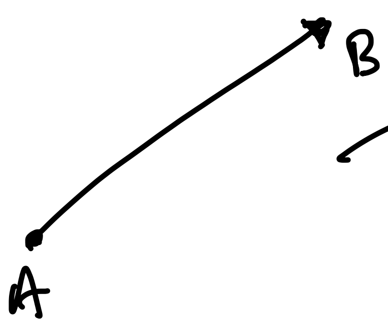
Vectors and Coordinate systems.

• A vector is an entity endowed with a magnitude and a direction.

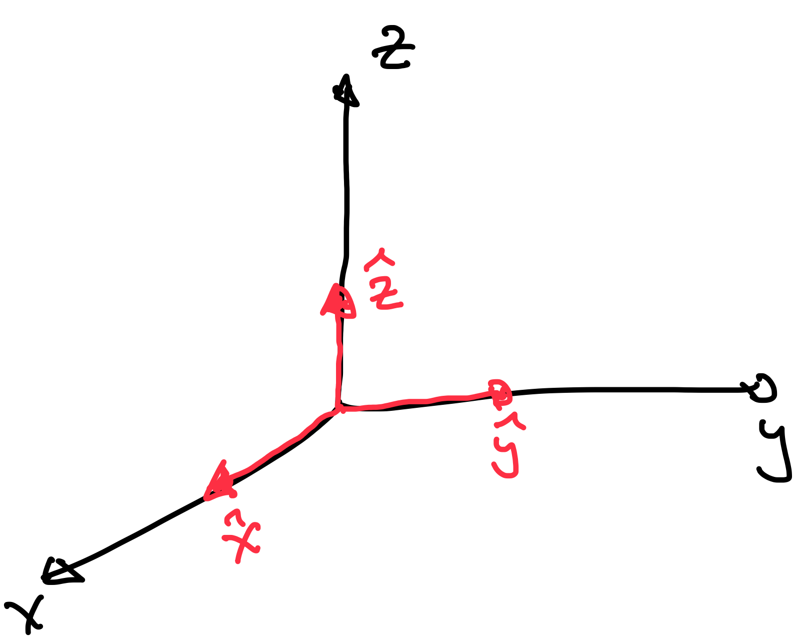
Eg.



Vector pointing from $A \to B \implies \overrightarrow{AB} \equiv \vec{v}$

• In this course I will use the notations $\vec{v}$, $\overset{\rightharpoonup}{v}$ or $\mathbf{v}$ to describe a vector.

• To get the directions of a vector, we first need to define a coordinate system.

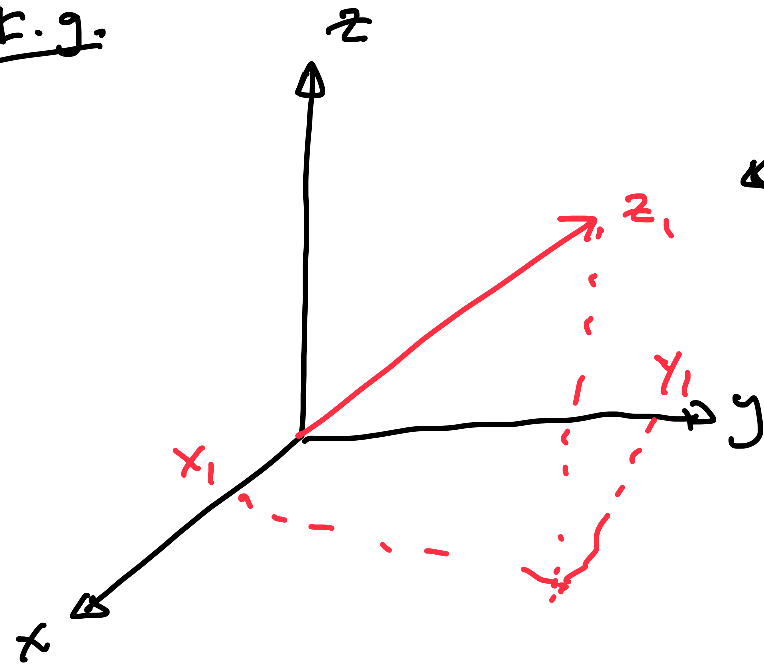


Unit vectors.

$\hat{x}, \hat{y}, \hat{z}$ are unit vectors, i.e. magnitude is one unit.

or $|\hat{x}| = |\hat{y}| = |\hat{z}| = 1$

E.g.



The vector is thus described by:

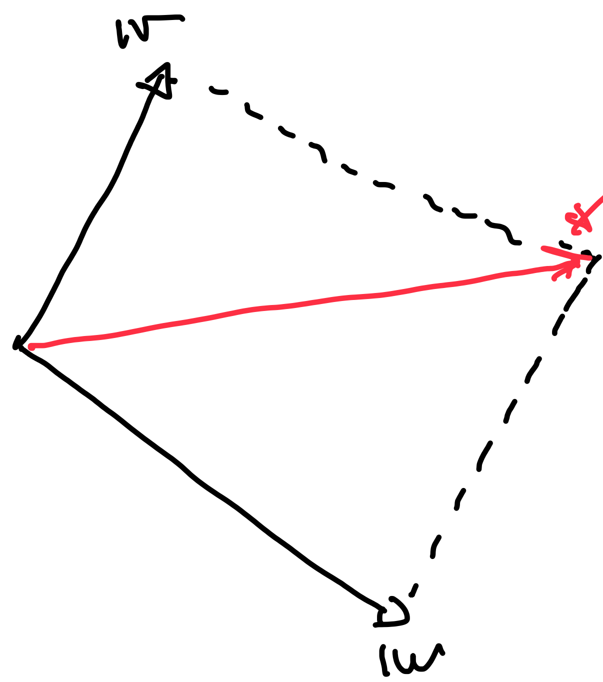
$$
\begin{aligned}
\mathbf{v} &= \langle x_1, y_1, z_1 \rangle \\
&= x_1\hat{i} + y_1\hat{j} + z_1\hat{k} \\
&= x_1\delta_x + y_1\delta_y + z_1\delta_z \\
&= \underbrace{x_1\delta_1 + y_1\delta_2 + z_1\delta_3}_{\text{Many ways to write out vectors....}}
\end{aligned}
$$

- The magnitude (length) of a vector is simply: $|\mathbf{v}| = \sqrt{x_1^2 + y_1^2 + z_1^2}$

- Note that for the more general case, where the vector does <u>not</u> start at the origin, is:

$$|\mathbf{v}| = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}$$

Vector addition.



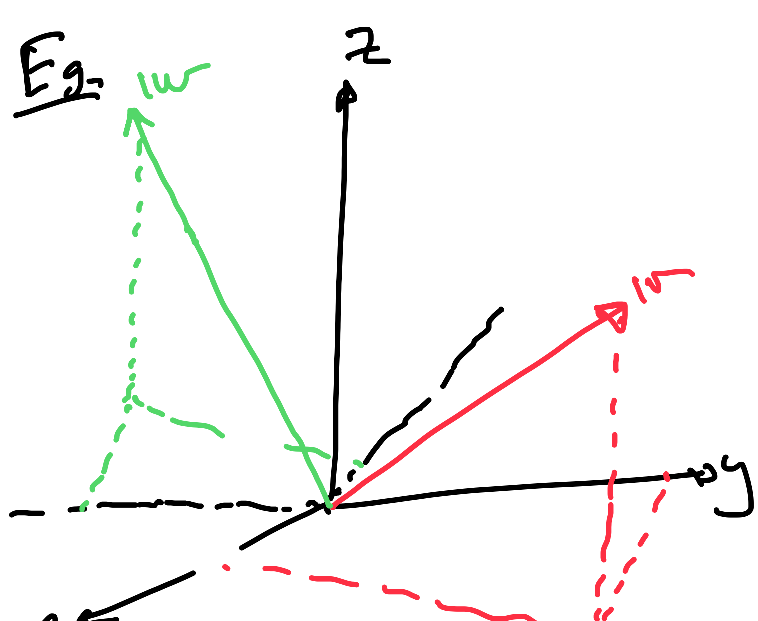
$\mathbf{v} + \mathbf{w} = (v_x + w_x)\,\delta_x +$
$(v_y + w_y)\,\delta_y +$
$(v_z + w_z)\,\delta_z.$

Vector multiplication.

- The "dot product" of two vectors results in a scalar that describes how parallel the two vectors are to eachother.

Definition : $\mathbf{v} \cdot \mathbf{w} = \sum_{i=1}^{3} v_i w_i = v_1 w_1 + v_2 w_2 + v_3 w_3$

Eg.



$\mathbf{v} = \langle 1, 1, 1 \rangle.$

$\mathbf{w} = \langle -1, -1, 2 \rangle.$

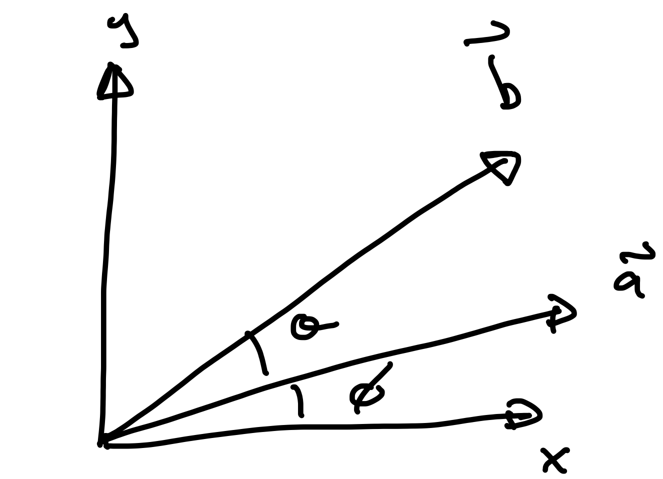
Hence,

$\mathbf{v} \cdot \mathbf{w} = 0$, and

$\mathbf{v}$ is orthogonal to $\mathbf{w}$.

Try at home

Prove that $\vec{a} \cdot \vec{b} = |a||b| \cos\theta$.



Hint $\Rightarrow$ $a_x = |a| \cos\theta$

$a_y = |a| \sin\theta$

Cross product.

- The cross product between two vectors results in a vector that is orthogonal to both vectors.

Definition : $\mathbf{v} \times \mathbf{w} = \begin{vmatrix} i & j & k \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{vmatrix}$

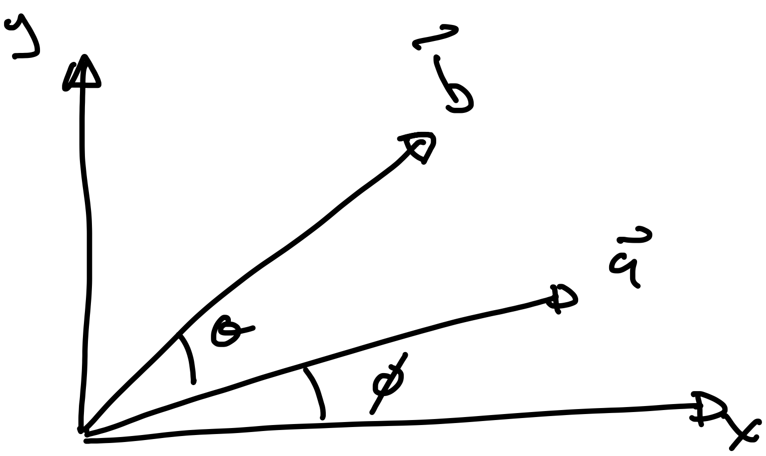
$= (v_y w_z - v_z w_y)\, \delta_1 -$

$(v_x w_z - v_z w_x)\, \delta_2 +$

$(v_x w_y - v_y w_x)\, \delta_3$

Try at home

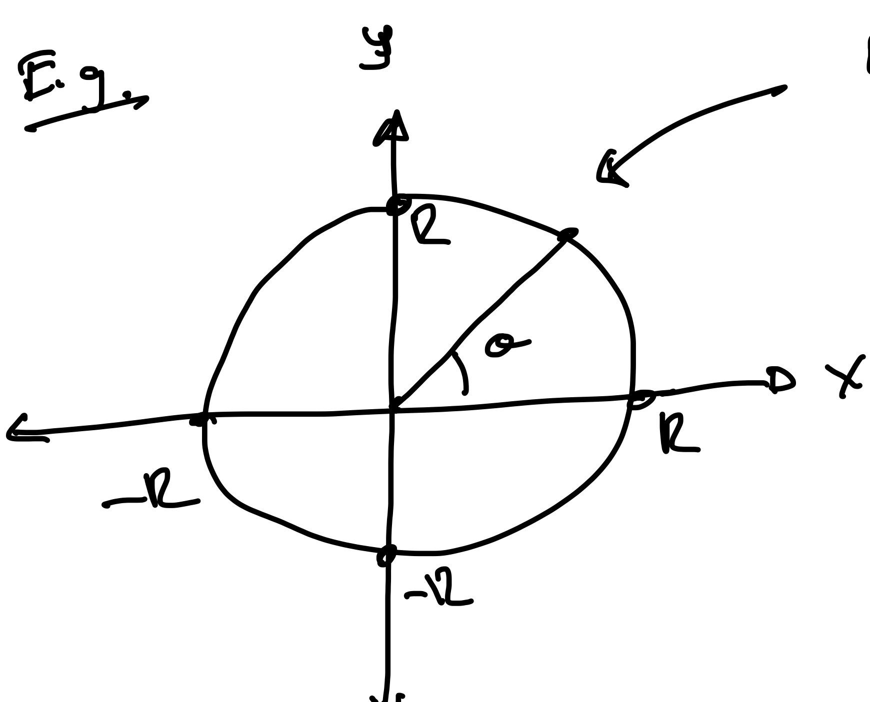
Show that $\vec{a} \times \vec{b} = |a||b| \sin\theta$



Curves:

Our objective is to describe the position along a curve with a vector function.

E.g.



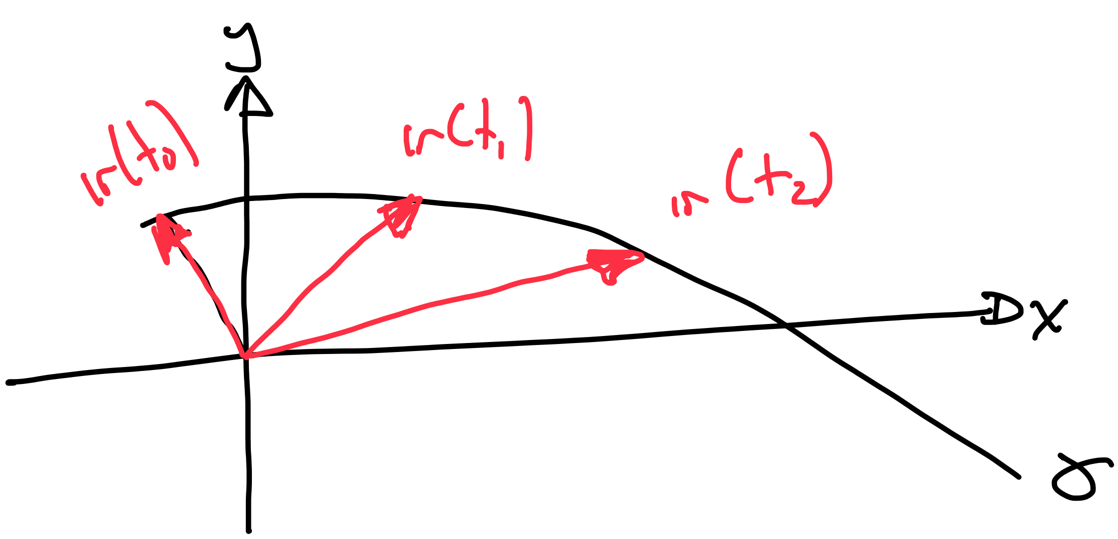
We know that for a circle of Radius "R", the position along the curve is.

$$\boxed{\begin{aligned} x &= R\cos\theta \\ y &= R\sin\theta \end{aligned}}$$

Hence, we can write a vector valued function that points everywhere along the circle, ie.

$$\mathbf{r}(\theta) = \langle x(\theta), y(\theta) \rangle$$
$$= \langle R\cos\theta, R\sin\theta \rangle.$$

- We can think of θ as a linear variable that describes the position along the curve.



we can do the same thing for arbitrary curves, where you can think of "t" as time. This boils down to:

$$\gamma := \{ (x(t), y(t)) \in \mathbb{R}^2 : \mathbf{r}(t) = \langle x(t), y(t) \rangle \}$$