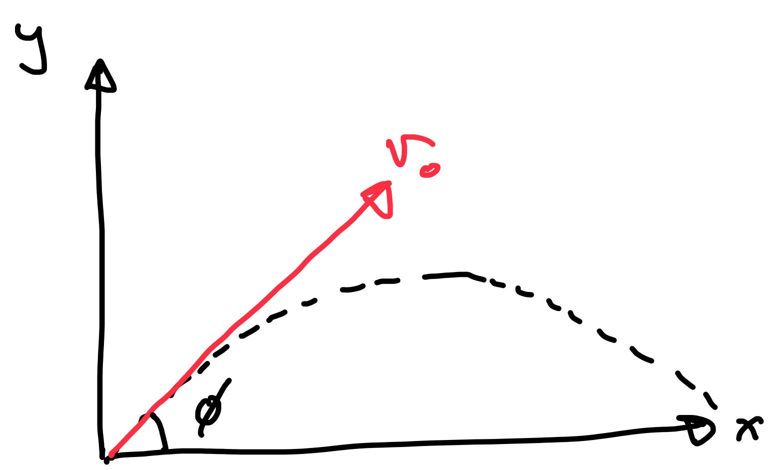
Application Problems.

- Let's now use parametrizations, vector to curves; curve formula to solve real world problems.

E.g.

Firing a projectile.



- Let's say I fire a projectile from $(x(0), y(0)) = (0,0)$ at an initial speed $|v_0|$ and at an angle ϕ. When does it hit the ground?

→ If we ignore drag forces (e.g. viscous forces from the air) the only force acting on the projectile is gravity, $g = -9.81\ m/s^2$. or $F_g = mg$.

→ Let's first start by writing out the acceleration due to gravity, i.e.

$\mathbf{r}''(t) = \langle 0, -9.81 \rangle$.

Integrating once, we get:

$\mathbf{r}'(t) = \langle c_1, -9.81t, c_2 \rangle$, which describes the velocity of the projectile along it's path.

We solve for c_1 & c_2 by applying the initial velocity condition.

$\mathbf{r}'(0) = \langle c_1, c_2 \rangle = \langle |v_0| \cos\phi, |v_0| \sin\phi \rangle$,

Hence,

$\mathbf{r}'(t) = \langle |v_0| \cos\phi, |v_0| \sin\phi - 9.81t \rangle$.

Integrating once more, we get:

$\mathbf{r}(t) = \langle |v_0| t \cos\phi + c_3, |v_0| t \sin\phi - \frac{9.81 t^2}{2} + c_4 \rangle$.

Now, with $(x(0), y(0)) = (0, 0)$, we get.

$$\boxed{\mathbf{r}(t) = \langle |v_0| t \cos\phi, |v_0| t \sin\phi - \frac{9.81 t^2}{2} \rangle}$$

To find when the projectile hits the ground, we set: $y(t) = 0$, and solve for t.

$$\frac{9.81\, t^2}{2} = |v_0|\, t \sin\phi \Rightarrow \boxed{t = \frac{2|v_0| \sin\phi}{9.81}}$$

Question

- At what angle should the projectile be fired to maximize horizontal distance?

$$x(t) = |v_0|\, t \cos\phi$$

$$= \frac{2|v_0|^2 \sin\phi \cos\phi}{9.81}$$

we need to maximize this expression.

Recall that: $2\sin\phi\cos\phi = \sin(2\phi)$.

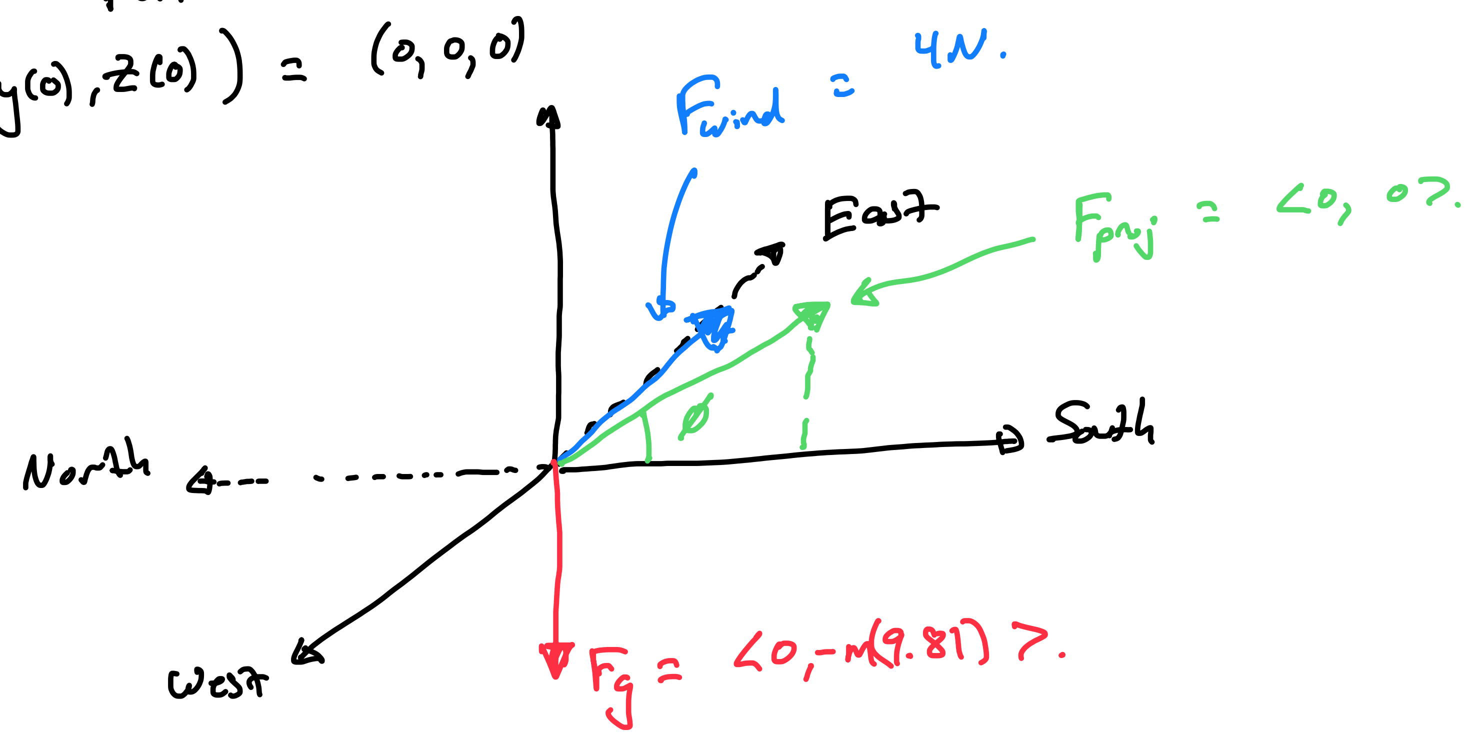
Hence,

$$x(t) = \frac{|v_0|^2 \sin(2\phi)}{9.81} \quad \therefore \quad 2\phi = \pi/2 \quad \text{or} \quad \boxed{\phi = \pi/4}$$

Ex.

A 1kg ball is fired South into the air at $|v_0| = 30$ m/s at $\phi = 30°$ to the ground. A west wind of $F = 4$ N pushes the ball East. Where does the ball land and at what speed?

Assume that the ball was fired from $(x(0), y(0), z(0)) = (0, 0, 0)$



→ As we've already done, we start by writing out the forces acting on the ball.

$$r''(t) = \langle x''(t), y''(t), z''(t) \rangle.$$

$$= \langle \frac{F_{wind}}{mass}, 0, F_g/mass \rangle$$

$$= \langle \frac{-4}{1kg}, 0, -9.81 \rangle.$$

Integrating, we get:

$$\mathbb{r}'(t) = \langle -4t + c_1,\ c_2,\ -9.81t + c_3 \rangle.$$

Apply the initial velocity condition,

$$\mathbb{r}'(0) = \langle c_1, c_2, c_3 \rangle = \langle 0, |v_0|\cos\phi, |v_0|\sin\phi \rangle.$$

$$= \langle 0,\ 15\sqrt{3},\ 15 \rangle.$$

$$\mathbb{r}'(t) = \langle -4t,\ 15\sqrt{3},\ 15 - 9.81t \rangle.$$

Now, integrate again....

$$\mathbb{r}(t) = \langle -2t^2 + c_4,\ 15\sqrt{3}t + c_5,\ 15t - \frac{9.81t^2}{2} + c_6 \rangle.$$

Apply I.C.,

$$\mathbb{r}(0) = \langle 0, 0, 0 \rangle = \langle c_4, c_5, c_6 \rangle.$$

Now, let's find when the ball hits the ground.

We solve for $z(t) = 0$.

ie, $15t = \frac{9.81t^2}{2} \Rightarrow t_{ground} = \frac{30}{9.81}$ sec.

Subing this into $\mathbb{r}(t)$ & $\mathbb{r}'(t)$, and we get the velocity & position of the ball when it hits the ground.