Zombie Prevalence and Bank Health: Exploring Feedback Effects

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Abstract. This paper investigates feedback effects between bank health and zombie firms—financially distressed firms receiving subsidized credit. The literature focuses on how banks create zombies, overlooking zombies' impact on bank health. Using Spanish firm-bank data (2005-2014), we document a vicious cycle: lower bank capital ratios increase zombie activity in served industries, while higher zombie prevalence undermines bank capital. We link this to a previously unexplored mechanism where banks respond appropriately to observable financial distress through higher provisioning, but overlook risks from relationship borrowers receiving subsidized rates. Our findings suggest that this feedback stems from distress combined with interest rate subsidies.

Keywords. Zombie Lending; Bank-Firm-Industry Feedback; Capital Misallocation; Networks; Fixed Effects; Cross-Sectional Dependence

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1. Introduction

The practice of extending credit to "zombie firms"—financially distressed borrowers that are artificially kept alive through bank forbearance—has been linked to protracted periods of economic stagnation following major shocks, and often coupled with loose regulatory environments. First documented by Peek and Rosengren (2005) and Caballero, Hoshi, and Kashyap (2008) in the context of Japan's "lost decade", the zombie lending phenomenon has attracted renewed attention in light of Europe's slow recovery after the financial and sovereign debt crises, and more recently, following many governments' efforts to keep firms supplied with credit after the COVID-19 pandemic.[1]

The literature has focused on how banks give rise to zombie firms, paying little attention to the reciprocal impact of zombies on bank health.[2] Yet this relationship is likely bidirectional: weak banks may engage in zombie lending to avoid realizing losses, while, at the same time, zombie firms may also harm banks by eroding loan portfolio quality. Feedback between the real and financial sectors, if present, could amplify the negative impact of zombie lending. Understanding whether such feedback exists is crucial for assessing the true economic costs of zombie lending.

In this paper, we investigate the feedback effects between bank capital ratios and zombie prevalence. To the best of our knowledge, ours is the first paper to study the reverse channel from zombie firms back to banks, alongside the commonly studied channel from banks to zombies. We develop an empirical model and estimation strategy uniquely suited for our context. Our approach addresses the inherent cross-sectional dependence between firm and bank outcomes that arises from the network of firm-bank relationships.

Our results show that significant feedback exists between zombie prevalence and bank capitalization. Lower bank capital ratios lead to increased zombie proliferation in served industries, while higher zombie prevalence subsequently undermines bank capital, creating a

[1]See Acharya, Crosignani, Eisert, and Steffen (2022) and Albuquerque and Iyer (2024) for recent surveys of the zombie literature.
[2]See Section 2.1 for a discussion of the literature documenting why banks extend credit to zombie firms.

vicious cycle. Quantitatively, a one-percentage-point decrease in bank capital ratios increases the zombie share in an industry by 3.3 percentage points, while a one-percentage-point increase in the share of zombie borrowers reduces a bank's capital ratio by 3 basis points.

For our analysis, we use detailed firm-bank matched data from Spain between 2005 and 2014. Our firm-level data comes from Bureau van Dijk's Orbis dataset, which we match to bank-level information from the Bankscope dataset. Our final sample covers 91 banks and over $152,000$ firms operating in 573 industries. We adopt the zombie definition from Acharya, Crosignani, Eisert, and Eufinger (2024), whereby a firm is considered a zombie if (i) it is financially distressed, meaning that it has above-median leverage and below-median interest coverage ratio, and (ii) it receives subsidized credit, meaning that it borrows at a rate lower than that of its most creditworthy industry peers.

The distinction between financial distress and subsidized lending is crucial for understanding our results. We find that while banks respond appropriately to observable financial distress through higher provisioning and better capital management, they appear to overlook the risks posed by relationship borrowers receiving subsidized rates. It is not financial distress alone that harms banks, but rather the combination of distress with interest rate subsidies that drives the destabilizing feedback mechanism we document.

An empirical investigation of bidirectional interactions between firms and banks presents methodological challenges that cannot be handled by the standard panel regression framework. The main challenge stems from the network dependence that naturally arises when shocks propagate between banks and firms through lending relationships. Two banks' outcomes can be simultaneously affected if a negative shock hits their shared borrowers. Similarly, the zombie status of two firms can be triggered by a negative shock to a bank that both firms borrow from. This network structure prevents us from assuming that bank and firm outcomes are independent and forces us to account for the cross-sectional dependence that standard panel methods ignore.

For tractability, we develop our methodology within the framework of fixed-effects linear panel regression models and show that it delivers valid inference despite complex cross-sectional dependence among bank and firm outcomes. Accommodating this dependence is crucial in our setting because it emerges naturally from the lending relationships between banks and firms. Such dependence is likely to arise in any framework that relies on firm-bank data and warrants explicit treatment, which has not received attention in the zombie literature. Regarding regression errors, we accommodate a flexible form of cross-sectional dependence by allowing errors to be correlated through past shocks.[3] We achieve this flexibility through a variant of the Helmert transform from Arellano and Bover (1995) for handling fixed effects and time effects. An additional advantage of the Helmert transform is that, under mild conditions, it enables us to identify the sign of regression coefficients even in the presence of measurement error in firms' zombie status.[4]

Our methodology is easy to use and well-suited to our empirical setting. Specifically, we show in simulations that our method exhibits stable finite sample properties and retains adequate power across a wide range of network densities. This feature is important in our context because several banks are connected to many firms, resulting in dense networks.

Our framework enables us to contribute to the literature by uncovering a previously unrecognized channel through which zombie firms are harmful: by inflicting damage on the very banks that extended them credit. While previous literature has extensively documented how zombie firms inhibit aggregate economic activity and crowd out healthy competitors, their impact on banks has been overlooked.[5] The feedback from zombie firms to banks that we

[3]We avoid network dependence in regression errors for two practical reasons. First, such specifications make estimators and inference highly sensitive to potential network misspecification. Second, network-dependent errors often yield overly conservative inference that discards valuable information, as statistical procedures typically assume worst-case scenarios where all linked variables are strongly correlated, substantially reducing the effective sample size. Given that some banks in our data connect to many firms, the resulting loss of statistical power could be severe.

[4]Any zombie definition that involves interest rates and/or interest payments is prone to measurement error because of the inability to observe counterfactual interest rates that firms would obtain in the absence of zombie lending practices by banks.

[5]See Section 2.3 for a discussion of the literature on the adverse real effects of zombie congestion.

document suggests that policies aimed at mitigating zombie lending may yield benefits not only for economic efficiency but also for financial stability.

Our paper is organized as follows. Section 2 positions our work within the literature on the relationship between zombie lending and bank performance. Section 3 describes the data, while Section 4 details how we measure zombie prevalence and bank outcomes. Section 5 introduces our econometric methodology and provides Monte Carlo simulation results on the finite sample performance of our proposed estimators. Our empirical results are presented and discussed in Section 6. Section 7 concludes. The Appendix provides an analysis of the effect of measurement error in the zombie classification, along with robustness checks to the empirical results and the formal assumptions underlying our econometric methodology. The proof of asymptotic validity is presented in the Supplemental Note.

2. Zombie Lending and Bank Capital: A Two-Way Relationship

In this section, we examine the theoretical foundations for bidirectional relationships between bank capital and zombie prevalence. The literature has extensively documented how bank incentives drive zombie lending decisions, but the reverse channel remains unexplored. We also review why zombie prevalence at the industry level, rather than just individual zombie firms, amplifies negative economic spillovers.

2.1. Why Banks Engage in Zombie Lending

One of the leading explanations in the literature for why banks keep zombie firms afloat involves bank capital constraints.[6] Caballero, Hoshi, and Kashyap (2008) is the first study

[6] While some papers show that zombie lending can occur even when banks are well-capitalized (Hu and Varas, 2021; Faria-e Castro, Paul, and Sánchez, 2024), or can be perpetrated by non-bank financial institutions (Favara, Minoiu, and Perez-Orive, 2024), these findings are based on evidence from the U.S. In Japan and Europe, on the other hand, zombies are typically tied to poorly capitalized banks. See Peek and Rosengren (2005) and Giannetti and Simonov (2013) for examples from Japan, and Blattner, Farinha, and Rebelo (2023) and Acharya, Crosignani, Eisert, and Eufinger (2024) for Europe. The latter study finds that in Europe, only 32% of zombie firm assets are linked to well-capitalized banks. Given this evidence, we focus on feedback between banks and zombies through the lens of bank capital.

to put forth the idea that banks extend zombie lending to avoid writing off existing capital. Bruche and Llobet (2014) propose a model of "gambling for resurrection", where weakly capitalized banks delay loss recognition by keeping insolvent borrowers alive rather than realizing immediate losses. This limited liability distortion is particularly relevant in bank-dependent economies like Spain, where banks know that refusing to evergreen loans will likely force firms into bankruptcy, while continued lending offers a chance for recovery.

Policy choices, particularly regulatory forbearance towards banks, can also inadvertently encourage zombie lending. Acharya, Lenzu, and Wang (2021) note that large negative shocks are increasingly met with unconventional monetary policy which often involves lenient bank regulation. They show in a model of heterogeneous firms and banks that such aggressive accommodative measures create a diabolical sorting where low-capital banks extend loans to low-productivity firms, and that policies designed to prevent short-term recessions can get stuck into excessive forbearance as zombie firms crowd out healthier competitors. Acharya, Borchert, Jager, and Steffen (2021) document that during the 2008–2009 financial crisis, fiscally constrained Eurozone governments "kicked the can down the road" by providing bank guarantees rather than full recapitalizations, prompting undercapitalized banks to engage in zombie lending.

2.2. Why Zombies Can Affect Bank Capital

While the literature cited above has focused on how banks' incentives drive zombie lending, the reverse relationship—how zombie lending affects bank performance—has received little attention. Yet this feedback channel is crucial for understanding the true cost of zombie firms.

To visualize how feedback can occur, it is useful to break down a bank's options once one of its borrowers shows visible signs of financial distress. Table 1 sketches out a simple scenario in which the distressed firm ultimately fails to recover, illustrating how zombie lending can backfire. Path A represents what a healthy bank would do: promptly classify the loan as non-performing, provision for expected losses and suffer an immediate capital loss while

TABLE 1. Timeline of Bank Balance Sheet Effects

T=0: Firm shows distress signals	
Path A: Bank reacts normally	
T=1	Classify as non-performing loan (NPL) → No immediate balance sheet impact → Regulatory scrutiny increases
T=2	Must set aside loan loss provisions (LLP) → Reduces net income → Lowers capital ratio, all else equal → May trigger regulatory action
T=3	Orderly resolution → Incur net charge-offs = Loan value - Recovery → Capital impact partially absorbed by prior provisions
Path B: Bank engages in zombie lending	
T=1	Extend subsidized credit → Maintains "performing" loan status → Continues receiving interest income (albeit low) → Delays loss recognition
T=2	Minimal/reduced provisioning → Preserves capital ratio → But accumulates hidden losses
T=3	Delayed resolution → Incur larger net charge-offs (no prior LLP buffer, lower recovery rate) → Sudden negative capital ratio shock

pursuing orderly resolution. Path B shows how banks with distorted incentives can postpone loss recognition by maintaining loans as performing despite clear distress signals. Doing so allows banks to preserve reported capital ratios and avoid regulatory intervention in the near term, but ultimately exacerbates capital losses if the firm fails to recover.

Our timeline analysis illustrates how zombie lending can create a negative feedback loop. The accumulation of zombie loans hurts bank capitalization through subsidized rates, delayed loss recognition, and reduced lending capacity to productive firms. This weaker capitalization then incentivizes more zombie lending, potentially amplifying the phenomenon.

2.3. Why Zombie Prevalence Matters

Keeping zombie firms afloat can have adverse spillover effects on the rest of the economy by inhibiting the movement of resources from less productive to more productive uses. These effects become amplified when zombies cluster within sectors, as their agglomeration creates sclerotic business environments with high degrees of resource misallocation, which have been shown to contribute to sizable losses in aggregate productivity (Hsieh and Klenow, 2009; Midrigan and Xu, 2014; Gopinath, Kalemli-Ozcan, Karabarbounis, and Villegas-Sanchez, 2017).

Caballero, Hoshi, and Kashyap (2008) show that zombie congestion reduces profits for healthy firms and depresses job creation and productivity in affected industries. More recent papers have found evidence that industries with a high degree of zombie congestion experience reduced levels of economic activity (Acharya, Eisert, Eufinger, and Hirsch, 2019), productivity-enhancing reallocation (Adalet McGowan, Andrews, and Millot, 2018), as well as innovation and R&D intensity (Schmidt, Schneider, Steffen, and Streitz, 2020). Acharya, Crosignani, Eisert, and Eufinger (2024) further show that industries with a high degree of zombie prevalence experience lower firm entry and exit, capacity utilization, and inflation.

These industry-level distortions underscore that zombie prevalence, as opposed to just the presence of individual zombie firms, is an important driver of macroeconomic outcomes, which is why we use it as one of our key outcomes of interest.

3. Data

We collect detailed data from Spain between 2005 and 2014 which links narrow sectors of real economic activity to the credit institutions providing them with external funding. Our industry-level data is derived from annual firm-level balance sheets from Bureau van Dijk's (BvD) Orbis dataset. The coverage of the firm-level data is comprehensive: firms in the sample account for 69-82% of Spanish gross output in the period 2005-2012 and the share of activity accounted for by small, medium and large firms closely resembles that observed in aggregate data.[7] We drop firm-year observations with non-positive values for total assets, tangible fixed assets, and number of employees as well as entries with negative liabilities

[7]See Kalemli-Özcan, Sørensen, Villegas-Sanchez, Volosovych, and Yeşiltaş (2024). 2006 gross output shares for small (< 19 employees), medium ($20-249$ employees) and large (> 250 employees) firms are, respectively, $(0.22, 0.39, 0.40)$ in the Orbis data and $(0.21, 0.38, 0.41)$ if the aggregate data from Eurostat.

and net worth. We drop firms in the financial sector (NACE Rev. 2 codes 64-66) and only keep observations for which basic accounting identities are satisfied.[8] Nominal quantities are deflated using industry-specific GDP deflators from Eurostat.

In addition to balance sheet characteristics, the firm-level data also contains information on firms' status of activity in a given year. This is used to determine if and when a firm exits the market, which we define as the event in which a firm is being dissolved or is undergoing bankruptcy proceedings.[9] We view our definition of exit as conservative because it does not take into account other reasons for which firms can show up as inactive in the data, which include being part of a "(de-)merger" as well as "unknown" reasons.

In order to match firms to their banks, we exploit the variable called *banker*, which reports the names of up to ten credit institutions with which the firm has a relationship.[10] We take the fact that a firm reports the name of a bank to also mean that the bank lends to the firm, an assumption commonly made in the literature on firm-bank relationships.[11] The banker variable does not include a time stamp, meaning that we cannot determine when a lending relationship started or whether it changed over time. This shortcoming is mitigated by evidence that lender-borrower relationships tend to be stable over the business cycle.[12]

We match the bank names reported in the firm-level data with bank financial statements from BvD's Bankscope dataset.[13] We exclude credit institutions specializing in consumer credit, such as credit card and leasing companies, as well as private security and asset management companies. We construct our yearly panel of banks so as to maximize both the number of banks and the number of time periods, as both dimensions are important for our

[8]The criteria are as in Gopinath, Kalemli-Ozcan, Karabarbounis, and Villegas-Sanchez (2017).

[9]Note that the timing of firm exit is somewhat imprecise. For most exiting firms there is a gap of several years between the last valid observation and the year when the status changed to inactive. In those cases, we define the exit year to be the year immediately following the firm's last observation. This assumption is also made in other papers looking at firm exit, e.g. Aghion, Bergeaud, Cette, Lecat, and Maghin (2019).

[10]This information is included in our firm-level database, but the original source is KOMPASS.

[11]See, for example, Kalemli-Ozcan, Laeven, and Moreno (2022) and Laeven, McAdam, and Popov (2018) who also infer a lending relationship from the same data source.

[12]Giannetti and Ongena (2012) look at different vintages of the banker variable and find it to be very persistent. Chodorow-Reich (2014) finds consistent evidence using different data from the U.S.

[13]The matching is done based on names for lack of a common identifier in the two data sources.

methodology. To do so, we prioritize unconsolidated accounts over consolidated ones where possible, while making sure to avoid double-counting issues.[14] By using unconsolidated accounts we also avoid the possibility that variation at the individual bank level is lost at the consolidated level.

After cleaning the firm data according to the procedure outlined above, we are left with over 152,000 firms operating in 573 four-digit NACE Rev. 2 industries. On the bank side, we have information on 91 banks. The number of cross-sectional units underlying the main results in Section 6 is lower, firstly, because not all firms are matched to banks covered by our data, and secondly, because our estimation procedure requires strongly balanced panels. To mitigate the loss of observations, we interpolate gaps in our bank-level variables of interest of up to one year.

4. Zombie Definitions and Capital Ratios

This section describes the construction of our key variables for analyzing the relationship between zombie prevalence and bank capital and presents stylized facts on firm characteristics, transition dynamics, and bank-firm-industry relationships.

4.1. Measuring Zombie Prevalence

We adopt the zombie definition from Acharya, Crosignani, Eisert, and Eufinger (2024), which captures two characteristics: financial distress and access to subsidized credit. To be financially distressed, a firm must have a leverage ratio above the industry median and an interest coverage ratio (ICR) below the industry median. A firm is considered to receive subsidized credit if it borrows at rates lower than those of AAA-rated industry peers.

We measure leverage as total debt over total assets, and define the ICR as earnings before interest and taxes (EBIT) divided by total interest payments. To avoid misclassification, we compare two-year averages of these ratios to their respective industry medians rather than

[14] We follow the steps outlined in Duprey and Le (2016) to create consistent time series.

using single-year values. Since we do not directly observe individual loan terms or credit ratings, we follow Acharya, Crosignani, Eisert, and Eufinger (2024) in constructing proxies for the interest subsidy component. We infer each firm's average interest rate from the ratio of total interest payments to total debt. To establish the AAA benchmark, we classify firms as AAA-rated when their ICR exceeds 8.5.

The literature has put forth a variety of zombie definitions. Some definitions only include the financial distress component, in part because the credit subsidy component is harder to measure.[15] Even when interest rates are directly observed, banks can show forbearance through other channels, such as extended amortization schedules or covenant waivers. Recent empirical evidence, however, strongly supports using both components. Acharya, Crosignani, Eisert, and Steffen (2022) systematically compare various zombie definitions and document that only those incorporating interest rate subsidies successfully detect economic inefficiencies, such as competitive distortions and credit misallocation. We therefore use both components in our definition.

Our empirical results reinforce the choice of zombie definition, confirming that financial distress and interest subsidies capture distinct firm characteristics. Accounting for both dimensions is crucial to understanding the feedback between banks and zombie prevalence that we document.

While our definition captures the key economic features of zombie lending, measurement challenges remain. Our zombie prevalence measure is inevitably subject to measurement error due to our inability to observe the counterfactual interest payments for firms not receiving zombie credit from banks. Nevertheless, our definition is designed to be conservative. In Appendix A.1, we show that under mild conditions, firms classified as zombies are likely to be true zombie firms, though some actual zombie firms may be misclassified as non-zombies. Importantly, Appendix A.1.2 presents conditions under which we can still identify the sign

[15]For examples of zombie definitions relying on firms' financial health alone, see Peek and Rosengren (2005), Adalet McGowan, Andrews, and Millot (2018), Banerjee and Hofmann (2018), Schivardi, Sette, and Tabellini (2020), Bonfim, Cerqueiro, Degryse, and Ongena (2023). There are also papers which only use the credit subsidy component: Caballero, Hoshi, and Kashyap (2008), Giannetti and Simonov (2013).

TABLE 2. Firm characteristics by status

	Mean			Median		
		Distressed			Distressed	
	Healthy (1)	Non-zombie (2)	Zombie (3)	Healthy (4)	Non-zombie (5)	Zombie (6)
Firm age	14.933	14.590	13.613	14.000	13.000	12.000
Total assets (million euro)	3.398	2.581	3.320	0.764	0.725	0.790
Sales (million euro)	2.795	1.813	1.874	0.649	0.531	0.340
Number of employees	17.365	14.330	14.805	6.000	6.000	5.000
Return on assets	0.028	−0.018	−0.019	0.020	0.002	0.000
EBITDA / assets	0.091	0.054	0.028	0.077	0.058	0.033
Leverage ratio	0.216	0.457	0.548	0.161	0.434	0.537
Interest coverage ratio	6.980	0.118	−0.814	2.976	1.025	0.809
Debt service capacity	1.080	0.167	0.063	0.419	0.129	0.060
Interest rate	0.108	0.086	0.031	0.058	0.067	0.030
Firms per year (average)	152,604	38,417	24,390	152,604	38,417	24,390

Notes: This table reports mean and median values of firm characteristics by financial distress and zombie status over the sample period. Distressed non-zombie firms satisfy the financial distress criteria (two-year average leverage above industry median and two-year interest coverage ratio below industry median) but do not receive subsidized credit (interest rate below AAA-rated industry peers). Zombie firms meet both the financial distress and subsidized credit criteria. Total assets and sales are deflated using Eurostat deflators. Return on assets is net income divided by total assets. EBITDA/assets is earnings before interest, taxes, depreciation and amortization divided by total assets. Leverage ratio is total debt (loans plus long-term debt) divided by total assets. Interest coverage ratio is EBIT (earnings before interest and taxes) divided by total interest payments. Debt service capacity is EBITDA divided by total debt. Interest rate is total interest payments divided by total debt.

of the coefficients of zombie prevalence and bank outcomes in our regression model, despite the presence of measurement error.

Table 2 compares firm characteristics across healthy, financially distressed non-zombie, and zombie firms. Zombie firms are similar to healthy firms in terms of age and total assets, though they are somewhat smaller in terms of sales and employment. Regarding profitability measures, both zombie and distressed non-zombie firms show similarly negative average returns on assets, while healthy firms have positive returns. However, zombies generate much lower earnings relative to assets than distressed non-zombies.

Financial measures show even larger differences. While both zombie and distressed non-zombie firms have high leverage compared to healthy firms, zombies' distress is most visible in debt servicing metrics. Average interest coverage ratios drop significantly from healthy firms

TABLE 3. Transition probabilities

	Healthy $_{t+1}$	Distressed Non-zombie $_{t+1}$	Distressed Zombie $_{t+1}$	Exit $_{t+1}$	Total
Healthy $_t$	86.74	6.29	4.02	2.95	100.00
Distressed Non-zombie $_t$	21.56	62.52	10.64	5.28	100.00
Distressed Zombie $_t$	20.24	21.87	53.34	4.54	100.00
Total	67.81	17.98	10.66	3.54	100.00
	Healthy $_{t+2}$	Distressed Non-zombie $_{t+2}$	Distressed Zombie $_{t+2}$	Exit $_{t+2}$	Total
Healthy $_t$	80.99	10.36	5.78	2.87	100.00
Distressed Non-zombie $_t$	33.31	50.73	11.01	4.95	100.00
Distressed Zombie $_t$	33.43	23.78	38.85	3.94	100.00
Total	67.57	18.77	10.32	3.34	100.00
	Healthy $_{t+3}$	Distressed Non-zombie $_{t+3}$	Distressed Zombie $_{t+3}$	Exit $_{t+3}$	Total
Healthy $_t$	78.14	12.75	6.45	2.65	100.00
Distressed Non-zombie $_t$	38.58	46.03	10.93	4.45	100.00
Distressed Zombie $_t$	40.14	24.92	31.61	3.34	100.00
Total	67.30	19.67	10.00	3.03	100.00

Notes: This table reports transition probabilities between firm statuses. Distressed non-zombie firms satisfy the financial distress criteria (two-year average leverage above industry median and two-year interest coverage ratio below industry median) but do not receive subsidized credit (interest rate below AAA-rated industry peers). Zombie firms meet both the financial distress and subsidized credit criteria. Exit means that the firm is dissolved, in bankruptcy or liquidation.

(strongly positive) to distressed non-zombies (barely positive) to zombies (negative). The median zombie firm has an interest coverage ratio below one, meaning its earnings are insufficient to cover interest expenses, while the median interest coverage ratio for other distressed firms is just above one. Debt service capacity follows a similar pattern, with zombies showing less than half the capacity of distressed non-zombies. Surprisingly, despite being in the worst financial condition, zombies pay much lower interest rates than both distressed non-zombies and even healthy firms. These patterns highlight the importance of distinguishing between different types of distressed firms.

Table 3 shows the transition probabilities between firm statuses for up to three years. Zombie status is somewhat persistent, with over half of these firms remaining in that category a year later and nearly 40% still classified as zombies after two years. As expected, distressed non-zombie firms are more likely to exit than healthy firms at any point in time. Most importantly, zombie firms exhibit lower exit rates than other distressed firms, which is consistent with the notion that banks keep zombie firms alive through continued credit provision, hindering market exit. The latter observation is consistent with the exit patterns documented by Alvarez, García-Posada, and Mayordomo (2023), who also use Spanish firm-bank data.

From Zombies to Zombie Prevalence. In our empirical analysis in Section 6, we aggregate individual zombie firms to construct measures of zombie prevalence at both the industry and bank levels. In addition to equal firm weights, we also employ asset-based weights at the sector level and debt-based weights at the bank level. The weights are chosen to represent economic importance: at the sector level, assets capture the relative size of zombies; at the bank level, debt represents the direct channel through which firms are tied to banks and provides a more accurate measure of bank exposure to zombie firms.

4.2. Capital Ratios and Bank Characteristics

Table 4 presents summary statistics and correlations with the capital ratio for key bank variables. Our primary bank measure, the capital ratio (total equity divided by total assets), averages 9.6% with substantial variation across banks. The correlations reveal expected patterns: better-capitalized banks tend to have higher profitability, as measured by the return on average assets, and lower reliance on deposits and short-term funding. Although the Tier 1 capital ratio is the primary measure used by banking regulators for capital adequacy requirements, we use the total capital ratio because it provides substantially more observations. Reassuringly, the two measures are highly correlated at 70%.

4.3. Bank-Firm-Industry Relationships

Table 5 summarizes the structure of bank-firm-industry relationships using 2005 data. The last two columns report average Herfindahl-Hirschman indices (HHI) of concentration calculated individually for each unit (firm, industry, or bank) and then averaged across all units

TABLE 4. Bank variables

Variable	N	Mean	Std. Dev.	P10	P50	P90	Corr. w/ CAPR
Capital ratio (CAPR)	639	0.096	0.055	0.050	0.086	0.138	1.000
Loan loss provisions / gross loans	639	0.011	0.108	0.000	0.006	0.018	−0.024
Deposits & short-term funding /assets	639	0.842	0.098	0.706	0.872	0.911	−0.275***
Return on average assets	639	0.465	1.138	0.050	0.450	1.050	0.287***
Non-performing loans / gross loans	432	0.037	0.085	−0.012	0.032	0.128	0.023
Tier 1 capital ratio	302	0.123	0.149	0.045	0.107	0.165	0.709***
Net charge-offs / gross loans	245	0.012	0.101	−0.009	0.001	0.017	0.352***

Notes: This table reports summary statistics and correlations with the capital ratio for bank-level variables. Capital ratio is total equity divided by total assets. Tier 1 capital ratio is high-quality regulatory capital divided by risk-weighted assets.

in each category. The last column weights relationships by economic importance using bank gross loans for the first two rows and firm debt for the last two rows.

From the firm perspective, banking relationships are highly concentrated. Firms use very few banks, averaging just 1.6 relationships with a median of one bank per firm. At the sector level, concentration is more moderate, with each sector served by an average of 13 banks. The industry-level HHI values of around 0.2 still indicate meaningful concentration, implying that some sectors are dominated by fewer, larger banks.

From the bank perspective, banks maintain highly diversified firm portfolios, as evidenced by very low average HHI values (0.098 unweighted, 0.007 weighted by firm debt). When we weight by firm debt size, concentration nearly disappears, indicating that bank lending is spread across many borrowers rather than concentrated in a few large ones. Banks also display broad sectoral coverage, serving 106 sectors on average, with similarly low debt-weighted HHI (0.020) showing that bank lending is well-diversified across sectors when adjusted by borrower importance.[16] Looking at the number of firms per bank, the average bank serves around 2,931 firms, but this distribution is extremely skewed—the median bank serves only 44 firms while some serve over 52,000.

The fact that a few large banks are connected to many firms—with one bank connected to as many as 40% of all firms—creates very dense firm-to-bank networks. While handling

[16] The limited degree of industry specialization by the banks in our sample is worth noting, as recent research has found that specialized banks are less likely to engage in zombie lending due to the negative congestion externalities that zombie firms impose on healthy industry peers (De Jonghe, Mulier, and Samarin, 2025).

TABLE 5. Bank-firm-industry relationships

	N	Mean	P10	P50	P90	Max	HHI	HHI (w)
Banks per firm	130,460	1.6	1	1	3	8	0.771	0.668
Banks per sector	569	13.2	6	11	23	53	0.194	0.238
Firms per bank	71	2,930.7	3	44	2,433	52,658	0.098	0.007
Sectors per bank	71	105.5	3	32	335	562	0.134	0.020

Notes: This table reports bank-firm relationship patterns using 2005 data. Each row shows the distribution of relationships from the perspective indicated, with individual HHI values calculated for each unit and then averaged. HHI denotes the unweighted Herfindahl-Hirschman index. HHI (w) uses the following weights: in the first two rows, banks are weighted by gross loans to capture lending capacity; in the last two rows, firms/industries are weighted by total debt to capture borrower importance.

cross-sectional dependence matters in any network setting, it is particularly important in our firm-to-bank analysis, which represents our methodological contribution to the literature. The density of these connections means we cannot reasonably ignore cross-sectional dependence, as standard methods that assume independent observations would lead to invalid inference. Our empirical approach addresses this by accounting for network-induced correlations while maintaining good power properties even in the presence of such dense networks, as we explain in detail in the following section.

5. Econometric Methodology

This section develops our empirical methodology for estimating bidirectional feedback effects between zombie prevalence and bank health. We present the econometric specifications, explain our estimation procedure, and demonstrate through Monte Carlo simulations that our method maintains good finite sample properties across varying network densities.

5.1. Building an Empirical Model

To investigate the feedback effects between zombie prevalence and bank health, we cannot rely on a standard linear panel regression framework because network connections between firms and banks create cross-sectional dependence among outcomes. Two banks' outcomes can be correlated through their exposure to common borrowers. Similarly, two firms' zombie status can be correlated if the firms borrow from the same bank. The same argument carries over to the industry level: two industries' shares of zombie firms can be correlated through exposure to common lenders. To address this issue, we exploit information on firm-bank relationships and construct separate regression models at the industry and bank levels in a way that takes into account the network dependence in the regressors.

5.1.1. **Bank Health to Industry-Level Zombie Prevalence.** We consider a zombie indicator $Z_{f,t}$ for firm f in year t, and let $B_{\ell,t}$ be a bank health measure such as the capital ratio of bank ℓ in year t. For each firm f, we define the firm-level bank exposure as

$$B_{f,t} = \frac{1}{n_{B,f}} \sum_{\ell \in N_{B,f}} B_{\ell,t},$$

where $N_{B,f}$ is the set of banks connected to firm f and $n_{B,f} = |N_{B,f}|$. We construct industry-level zombie prevalence as

$$Z_{i,t} = \sum_{f \in N_{F,i}} Z_{f,t} a_f,$$

where $N_{F,i}$ represents the set of firms in industry i and a_f is a firm-specific weight that sums to one across all firms in industry i. We consider two types of weights: (i) equal weights $a_f = 1/n_{F,i}$, where $n_{F,i} = |N_{F,i}|$, and (ii) asset-based weights, where a_f represents the time-averaged asset share of firm f in industry i.[17]

We adopt the following industry-level regression specification:

$$Z_{i,t} = \overline{X}'_{i,t-1} \alpha_1 + X'_{i,t-1} \tilde{\alpha}_1 + \delta_{bz} \overline{B}_{i,t-1} + u_{i,t}, \tag{5.1}$$

where industry-level bank health and control variables are defined, respectively, as

$$\overline{B}_{i,t-1} = \sum_{f \in N_{F,i}} B_{f,t-1} a_f \text{ and } \overline{X}_{i,t-1} = \sum_{f \in N_{F,i}} X_{f,t-1} a_f,$$

[17] We use time-averaged weights because the fixed effects panel model relies on within-group variation of outcomes over time, and time variation in asset sizes can arise for reasons unrelated to the zombie status of the firm.

and the error term is decomposed as

$$u_{i,t} = r_i + f_{bz,t} + \varepsilon_{i,t}. \tag{5.2}$$

The error term (5.2) is decomposed into three terms: r_i (industry-specific fixed effects), $f_{bz,t}$ (time effects), and $\varepsilon_{i,t}$ (industry-specific idiosyncratic time-varying shocks). Crucially, we allow the idiosyncratic shocks $\varepsilon_{i,t}$ to be correlated across industries through macro-shocks represented by $f_{bz,t}$.

We take $X_{i,t}$ to be the Herfindahl-Hirschman index of sales concentration in industry i at time t and $X_{f,t}$ to be the vector of sales growth and return on assets of firm f at time t. A priori, higher market concentration could affect zombie shares either positively, by distorting competition, or negatively, by favoring large firms with profit margins that make them more resilient to shocks. We expect sales growth to reduce zombie prevalence, as revenue increases provide firms with buffers that help them avoid falling behind on loan payments. Finally, we expect return on assets to negatively affect zombie prevalence, as it is an often-used proxy for firm profitability.

5.1.2. **Firm-Level Zombie Prevalence to Bank Health.** We first explain how we construct zombie prevalence at the bank level from firm-level information. It is important to distinguish between the effect of surviving zombies and those that exit at $t+1$, as they affect bank outcomes through different channels. Exiting firms reduce bank income and, all else equal, bank capital when their unpaid loans are written off, but this mechanical effect is not our main concern. Instead, we are primarily interested in whether surviving zombies undermine bank capital through continued underperformance. We define bank-level zombie exposure from exiting and non-exiting firms as

$$\overline{Z}^{\mathrm{E}}_{\ell,t} = \sum_{f \in N_{F,\ell}} Z^{\mathrm{E}}_{f,t} b_f \text{ and } \overline{Z}^{\mathrm{NE}}_{\ell,t} = \sum_{f \in N_{F,\ell}} Z^{\mathrm{NE}}_{f,t} b_f, \tag{5.3}$$

where $N_{F,\ell}$ denotes the set of firms connected to bank ℓ, and b_f is a firm-specific weight that sums to one across all firms connected to bank ℓ. We consider two types of weights: (i) equal

TABLE 6. Summary statistics

	N	Mean	Std. Dev.	P10	P50	P90
Panel A: Industry equation						
Zombie share	5,121	0.093	0.096	0.000	0.080	0.173
Bank capital ratio	5,121	0.070	0.008	0.061	0.070	0.079
Firm sales growth	5,121	0.000	0.170	−0.175	0.016	0.144
Firm return on assets	5,121	0.023	0.033	−0.009	0.022	0.055
Industry HHI	5,121	0.091	0.147	0.005	0.039	0.225
Panel B: Bank equation						
Bank capital ratio	639	0.096	0.055	0.050	0.086	0.138
Zombie firms	639	0.087	0.106	0.000	0.074	0.167
Financially distressed firms	639	0.285	0.199	0.000	0.265	0.500
Firm sales growth	639	−0.022	0.250	−0.194	−0.010	0.145
Firm return on assets	639	0.018	0.028	−0.008	0.015	0.050

Notes: This table shows summary statistics for variables in our industry-level (Panel A) and bank-level (Panel B) regressions, corresponding to equations (5.1) and (5.4), respectively. Firm-level variables are aggregated to the industry level (Panel A) and bank level (Panel B) using equal weights. A firm is considered financially distressed if, for the past two years, it has had above-median leverage and below-median interest coverage ratios. A firm is considered a zombie if it is financially distressed and receives an interest rate below that of AAA-rated industry peers. Bank capital ratio is total equity divided by total assets. Firm return on assets is net income divided by total assets. Industry HHI is the Herfindahl-Hirschman index of sales concentration.

weights $b_f = 1/n_{F,\ell}$, where $n_{F,\ell} = |N_{F,\ell}|$, and (ii) debt-based weights, where b_f represents the time-averaged debt share of firm f among the firms connected to bank ℓ. Then, the total zombie prevalence at the bank level is given by

$$\overline{Z}_{\ell,t} = \overline{Z}^{\mathsf{E}}_{\ell,t} + \overline{Z}^{\mathsf{NE}}_{\ell,t}.$$

We adopt the following bank-level regression specification:

$$\log B_{\ell,t} = \overline{X}'_{\ell,t-1}\beta_1 + \delta^{\mathsf{E}}_{zb}\overline{Z}^{\mathsf{E}}_{\ell,t-1} + \delta^{\mathsf{NE}}_{zb}\overline{Z}^{\mathsf{NE}}_{\ell,t-1} + \nu_{\ell,t}, \tag{5.4}$$

where bank-level control variables are defined as

$$\overline{X}_{\ell,t-1} = \sum_{f \in N_{F,\ell}} X_{f,t-1} b_f,$$

and the error term is decomposed as

$$\nu_{\ell,t} = r_\ell + f_{zb,t} + \eta_{\ell,t}.$$

The error term is decomposed into three terms: r_ℓ (bank-specific fixed effects), $f_{zb,t}$ (time effects), and $\eta_{\ell,t}$ (bank-specific idiosyncratic time-varying shocks). Similarly to the industry-level specification, we allow the idiosyncratic shocks $\eta_{\ell,t}$ to be correlated across banks through macro-shocks represented by $f_{zb,t}$.

We take $X_{f,t}$ to be the vector of financial distress indicators, sales growth, and return on assets of firm f at time t. The financial distress indicator equals one if the firm's two-year average leverage ratio is above the industry median and its two-year average interest coverage ratio is below the industry median. We control for financial distress separately because it is one of the components of our zombie definition (along with interest rate subsidies) and the two may affect banks differently. The coefficient on zombie prevalence therefore captures the additional impact of interest rate subsidies. We expect sales growth and return on assets to positively affect bank health, as growing, profitable firms are less likely to default and provide banks with stronger loan portfolios.

Table 6 presents summary statistics for the key variables used in our empirical analysis, showing that zombie firms comprise roughly 9% of firms at both the industry and bank levels, while bank capital ratios average around 7% and 10% in our industry and bank samples, respectively. Firm-level variables are averaged using equal weights. Table 13 in the Appendix reports the same statistics using the weights described above. When using asset-based weights at the industry level, zombie prevalence increases only slightly while bank capital ratios remain unchanged, suggesting that firms of different sizes are equally prone to being zombies and borrow from similarly capitalized banks. At the bank level, debt-weighted zombie exposure nearly doubles from 8.7% to 16.3%, suggesting that zombies, though relatively few, account for an outsized share of bank credit.

5.2. Estimation

Our main parameters of interest are δ_{bz} in (5.1) and $\delta_{zb} = (\delta_{zb}^{\mathsf{E}}, \delta_{zb}^{\mathsf{NE}})$ in (5.4), which capture the feedback effects between zombie prevalence and bank health. To estimate these parameters, we use the Helmert transform of Arellano and Bover (1995) on both regression models in order to eliminate time and fixed effects from the regressions. We then perform a method-of-moments estimation with an optimal weighting matrix.

Specifically, for each $t = 1, \ldots, T$ and $s = t, \ldots, T$, we define

$$h_{s,t} = \begin{cases} \sqrt{\dfrac{T-t}{T-t+1}}, & \text{if } s = t \\ -\dfrac{1}{\sqrt{(T-t)(T-t+1)}}, & \text{if } s = t+1, \ldots, T. \end{cases}$$

For any (ℓ, t)-specific variable $y_{\ell,t}$, we define $y^{\mathsf{H}}_{\ell,t}$ as

$$y^{\mathsf{H}}_{\ell,t} = \sum_{s=t}^{T} h_{s,t}(y_{\ell,s} - \overline{y}_s), \quad \overline{y}_s = \frac{1}{n_B} \sum_{\ell} y_{\ell,s}.$$

The transform of $(y_{\ell,t})$ into $(y_{\ell,t})^{\mathsf{H}}$ is called the within-group Helmert transform, which is the within-group variant of the Helmert transform proposed by Arellano and Bover (1995). For an (i, t)-specific variable $x_{i,t}$, we define $x^{\mathsf{H}}_{i,t}$ similarly by taking $\overline{x}_s$ to be the within-industry average of $x_{i,s}$, $i \in N_{F,i}$.

We define the parameter vectors

$$\gamma_{bz} = [\beta_1', \delta_{bz}]' \text{ and } \gamma_{zb} = [\alpha_1', \tilde{\alpha}_1', \delta_{zb}']'$$

along with the corresponding regressor vectors

$$V_{i,t-1} = [\overline{X}'_{i,t-1}, X'_{i,t-1}, \overline{B}_{i,t-1}]' \text{ and } W_{\ell,t-1} = [\overline{X}'_{\ell,t-1}, \overline{Z}^{\mathsf{E}\prime}_{\ell,t-1}, \overline{Z}^{\mathsf{NE}\prime}_{\ell,t-1}]'.$$

After the Helmert transform, we can write the regression models in (5.1) and (5.4) as

$$Z^{\mathsf{H}}_{i,t} = V^{\mathsf{H}\prime}_{i,t-1}\gamma_{bz} + u^{\mathsf{H}}_{i,t}, \quad i \in N_I, \text{ and} \tag{5.5}$$

$$(\log B_{\ell,t})^{\mathsf{H}} = W^{\mathsf{H}\prime}_{\ell,t-1}\gamma_{zb} + v^{\mathsf{H}}_{\ell,t}, \quad \ell \in N_B,$$

where N_B denotes the set of banks and N_I the set of industries. We estimate the parameters γ_{zb} and γ_{bz} using method-of-moment estimation with an optimal weighting matrix.

While this estimation method is intuitive, the asymptotic validity of this procedure has not been established in the literature, to the best of our knowledge. In Appendix A.3, we state assumptions and present a formal asymptotic validity result. Its proof is found in the Supplemental Note.

There are alternative approaches to remove fixed effects, such as within-group transform or first-differencing. However, the within-group Helmert transform is particularly useful in our setting for two reasons. First, it allows us to retain the martingale structure of the error terms $\eta_{i,t}$ and $\eta_{\ell,t}$, allowing them to be cross-sectionally correlated through past values of time effects and regressors. Second, under certain regularity conditions, it allows us to identify the sign of our parameters of interest δ_{bz} and δ_{zb} even in the presence of measurement error in the zombie classification, as we explain in detail in Appendix A.1.2.

5.3. Monte Carlo Simulations

5.3.1. **Data Generating Process.** Our simulation study adopts the following data generating process. First, we generate bipartite graphs according to the Chung-Lu configuration model (Chung and Lu, 2002). This allows us to model asymmetric cross-section sizes for simulated bank and industry data. We consider four configurations of cross-section sizes: $(n_B, n_I) = (50, 500)$, $(n_B, n_I) = (100, 500)$, $(n_B, n_I) = (200, 1000)$, and $(n_B, n_I) = (500, 5000)$. For every fixed pair (n_B, n_I), we generate two CL graphs and run all simulation specifications given those fixed graphs. Let N_i be the resulting in-neighborhood of cross-sectional unit i. Since our statistical inference is conditioned on the realized graphs, the stochastic nature of the graph generation is irrelevant both for asymptotic validity and finite sample performance. The summary statistics of the realized graphs are provided in Table 7.

We first generate fixed effects $r_i \sim N(1,1)$, $r_\ell \sim N(1,1)$, and time effects $f_{bz,t} \sim N(1,1)$, $f_{zb,t} \sim N(1,1)$ for each unit i representing industries, ℓ representing banks, and each period $0 \leq t \leq T$. We keep these effects fixed across simulation runs. In every simulation run, for each i, t, we generate a random vector $X_{i,t} \sim N(\mathbf{1}, I)$, where $\mathbf{1}$ is the vector of ones with

TABLE 7. Network characteristics

		CL 1		CL 5		CL 10		CL 50	
		d_{mx}	d_{mn}	d_{mx}	d_{mn}	d_{mx}	d_{mn}	d_{mx}	d_{mn}
$n_B = 50$	Bank	7	2.18	41	7.30	74	12.24	174	42.76
$n_I = 500$	Industry	11	2.02	31	5.99	50	10.93	50	33.09
$n_B = 100$	Bank	6	2.00	29	5.84	70	10.88	375	50.67
$n_I = 500$	Industry	8	2.01	27	6.08	48	10.96	100	45.51
$n_B = 200$	Bank	8	1.98	25	6.61	46	12.10	233	55.05
$n_I = 1000$	Industry	12	1.97	40	5.85	72	10.70	200	49.17
$n_B = 500$	Bank	9	1.94	38	5.85	79	10.60	344	49.36
$n_I = 5000$	Industry	12	1.96	45	5.94	91	10.90	415	50.43

Notes: This table presents the network characterstics for the simulation study. Networks are generated using the Chung-Lu configuration model. CL μ refers to mean expected degree of μ. d_{mx}, d_{mn} are realized maximum and mean degree, respectively. The mean degree vector as input to the configuration model is generated from an exponential distribution with mean μ.

dimension 3 and I the identity matrix. Analogously, we generate $X_{\ell,t}$ for banks. Letting N_i, N_ℓ, be the in-neighborhoods of industries i and banks ℓ, we construct $\overline{X}_{i,t} = \frac{1}{|N_i|}\sum_{\ell\in N_i} X_{\ell,t}$ as the diffusion-covariates, and analogously for $\overline{X}_{\ell,t}$. When $N_i = \emptyset$, we simply set $\overline{X}_{i,t} = 0$. Using outcome variables $Z_{i,t}, B_{\ell,t}$, we similarly construct $\overline{B}_{i,t} = \frac{1}{|N_i|}\sum_{\ell\in N_i} B_{\ell,t}$, and analogously for $\overline{Z}_{\ell,t}$. We generate i.i.d. error terms $\varepsilon_{i,t} \sim N(0,1)$, $\eta_{\ell,t} \sim N(0,1)$ for each i, ℓ, t. Recall from Section 5.1 that $u_{i,t} = r_i + f_{bz,t} + \varepsilon_{i,t}$ and $v_{\ell,t} = r_\ell + f_{zb,t} + \eta_{\ell,t}$. We set

$$Z_{i,0} = u_{i,0}, \text{ and } B_{\ell,0} = v_{\ell,0},$$

as initial outcomes for industries and banks, respectively. For every $1 \le t \le T$, we then generate

$$Z_{i,t} = \overline{X}_{i,t-1}\alpha_1 + X_{i,t-1}\tilde{\alpha}_1 + \overline{B}_{i,t-1}\bar{\delta}_{bz} + u_{i,t}, \text{ and}$$
$$B_{\ell,t} = \overline{X}_{\ell,t-1}\beta_1 + X_{\ell,t-1}\tilde{\beta}_1 + \overline{Z}_{\ell,t-1}\bar{\delta}_{zb} + v_{\ell,t}.$$

Here $\bar{\delta}_{zb}, \bar{\delta}_{bz}$ represent the outcome-diffusion parameter from industry to bank and bank to industry, respectively.

5.3.2. **Results.** We consider the two-sided testing problem under the null of $\delta_{h'h} = \bar{\delta}_{h'h}$, for $h, h' \in \{B, I\}$. Throughout, we keep all true coefficients $\bar{\delta}_{h'h}, \bar{\beta}_{h'h}, \bar{\alpha}_h$ set to equal (vectors of)

TABLE 8. Empirical rejection probabilities under the null hypothesis

		CL 1		CL 5		CL 10		CL 50	
		$T=10$	$T=20$	$T=10$	$T=20$	$T=10$	$T=20$	$T=10$	$T=20$
$n_B=50$	Bank	0.064	0.063	0.073	0.070	0.076	0.077	0.077	0.076
$n_I=500$	Industry	0.048	0.053	0.050	0.052	0.050	0.050	0.057	0.051
$n_B=100$	Bank	0.048	0.057	0.055	0.067	0.059	0.061	0.073	0.076
$n_I=500$	Industry	0.057	0.057	0.054	0.054	0.055	0.052	0.048	0.054
$n_B=200$	Bank	0.050	0.054	0.055	0.053	0.055	0.056	0.064	0.064
$n_I=1000$	Industry	0.051	0.050	0.053	0.048	0.054	0.055	0.054	0.049
$n_B=500$	Bank	0.054	0.05	0.050	0.054	0.055	0.051	0.053	0.053
$n_I=5000$	Industry	0.052	0.055	0.052	0.048	0.049	0.051	0.052	0.055

Notes: This table presents the empirical rejection probability under the null hypothesis that $\delta_{h'h} = \bar{\delta}_{h'h}$, for $h, h' \in \{B, I\}$. All simulations were run under the nominal level $\alpha = 0.05$. The Monte Carlo simulation number was 5,000.

1. We report empirical rejection probabilities and mean-length of confidence intervals from 5,000 simulations of the model.

Table 8 shows the empirical rejection probabilities under nominal level $\alpha = 0.05$, across different settings of the relative sample sizes of n_B and n_I, the number of the time periods T, and the network densities. The performance is good even in the setting with a stark asymmetry in (n_B, n_I). As expected, as the sample sizes n_B and n_I increase, the rejection probabilities approach the nominal level. As T increases, we observe some over-rejection for small cross-section sizes, an issue that is alleviated for larger cross-sectional sample sizes. Most notably, the finite sample validity shows its robust performance as network density increases. This suggests that our asymptotic inference performs well with this set of networks.

Table 9 looks into the mean length of the confidence intervals. As expected, the mean length of the confidence intervals shrinks as the sample sizes increase both in cross-section and time dimensions. This suggests that the law of large numbers works in this data generating process and the accuracy of the estimators improves with the sample sizes. However, we observe that the mean length increases with density of the networks. Such a reduction in power with denser networks has been observed in the literature (Kojevnikov, Marmer, and Song, 2021).

TABLE 9. Empirical mean length of confidence interval

		CL 1		CL 5		CL 10		CL 50	
		$T=10$	$T=20$	$T=10$	$T=20$	$T=10$	$T=20$	$T=10$	$T=20$
$n_B=50$	Bank	0.100	0.056	0.169	0.095	0.207	0.120	0.364	0.227
$n_I=500$	Industry	0.031	0.017	0.057	0.033	0.076	0.046	0.169	0.106
$n_B=100$	Bank	0.069	0.037	0.116	0.066	0.141	0.085	0.251	0.157
$n_I=500$	Industry	0.032	0.017	0.056	0.032	0.073	0.043	0.150	0.093
$n_B=200$	Bank	0.048	0.026	0.088	0.054	0.117	0.070	0.221	0.137
$n_I=1000$	Industry	0.021	0.012	0.039	0.023	0.052	0.031	0.105	0.066
$n_B=500$	Bank	0.027	0.015	0.050	0.030	0.068	0.041	0.137	0.085
$n_I=5000$	Industry	0.009	0.005	0.016	0.010	0.022	0.013	0.043	0.027

Notes: Average lengths of confidence intervals generated from standard error estimators in the simulations. All simulations were run under the nominal level $\alpha=0.05$. The Monte Carlo simulation number was 5,000.

6. Empirical Results

Our main results show significant feedback effects between zombie prevalence and bank capitalization, pointing to an amplification dynamic between these two phenomena. Specifically, lower bank capital ratios lead to a subsequent proliferation of zombie firms in the industries served by these banks, while at the same time, higher zombie prevalence among bank borrowers leads to deteriorating bank capitalization down the line.

To quantify these effects, we find that a one-percentage-point decrease in the capital ratios of the banks lending to an industry results in a 3.3 percentage-point increase in the share of zombie firms within that industry. Conversely, a one-percentage-point increase in the share of zombie borrowers leads to a 3-basis-point decline in a bank's capital ratio. Poorly capitalized banks foster zombie proliferation, which subsequently undermines bank capital, creating a vicious cycle.

Ours is the first paper to document the existence of significant feedback from zombie firms to bank capital, to the best of our knowledge. This finding is important for two key reasons. First, it identifies a previously unrecognized channel through which zombie firms inflict harm: by deteriorating bank capitalization and, consequently, threatening the stability of the broader banking system. Second, it shows that zombie lending practices are costly even from individual banks' perspectives. While banks often justify keeping zombie firms afloat to mask the balance sheet impact of impaired borrowers, our results show that this strategy at best provides only temporary protection. Eventually, these practices result in poorer capital ratios—precisely the outcome banks seek to avoid.

A crucial aspect of our analysis is the careful distinction between the two defining characteristics of zombie firms: financial distress and access to subsidized borrowing rates. As we explain in detail below, it is not financial distress alone that harms banks, but rather the combination of distress with interest rate subsidies that drives the feedback mechanism we document.

In what follows, we examine each of the two directions in the feedback chain and explain them in detail.

6.1. Effect of Bank Capital on Zombie Prevalence

Our main results on the effect of bank capitalization on industry-level zombie prevalence are shown in Table 10, which reports estimates from equation (5.1). The dependent variables are industry-level shares of zombie firms (columns 1-2) and financially distressed firms (columns 3-4).

The first column of Table 10 shows that industries exposed to worse-capitalized banks experience a significant increase in the share of zombie firms in the following year. The effect is economically significant: a one-percentage-point decrease in bank capital leads to a 3.3 percentage-point increase in zombie prevalence. The negative relationship is maintained when looking at asset-weighted zombie shares (column 2), albeit to a smaller degree. This finding is in line with much of the literature on zombie lending, which often finds that banks struggling to meet capital requirements are more likely to engage in zombie lending in order to avoid further damage to their balance sheet.

Turning to financial distress, which is a prerequisite for being a zombie, columns 3 and 4 of Table 10 show that it too increases in response to poorer bank capitalization. The proliferation in both zombie firms and financially distressed firms following a deterioration in

TABLE 10. Effect of bank capital ratios on zombie prevalence

	(1) Zombie	(2) Zombie, weighted	(3) Distressed	(4) Distressed, weighted
Bank capital ratio	-3.331^{**}	-2.661^{**}	-5.727^{***}	-3.545^{***}
	(1.321)	(1.098)	(1.947)	(1.281)
Firm sales growth	0.002	0.018	0.000	-0.003
	(0.019)	(0.014)	(0.022)	(0.017)
Firm return on assets	-0.496^{***}	-0.482^{***}	-0.743^{***}	-0.475^{***}
	(0.124)	(0.134)	(0.187)	(0.131)
Industry HHI	0.444^{**}	0.421^{*}	0.586^{**}	0.462^{*}
	(0.195)	(0.233)	(0.272)	(0.279)
Industry fixed effects	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes
Number of industries	569	569	573	573
Number of firms	130,460	130,460	132,932	132,932

Notes: This table shows results from estimating equation (5.1). The dependent variable is the industry-level share of zombie firms (columns 1-2) or financially distressed firms (columns 3-4). Columns 1 and 3 use equal weights, while columns 2 and 4 use asset weights based on each firm's average asset size over the sample period. A firm is considered to be financially distressed if, for the past two years, it has had above-median leverage and below-median interest coverage ratios. A firm is considered a zombie if it is financially distressed and receives an interest rate below that of AAA-rated industry peers. All regressors are industry-level averages using the same weighting scheme as the dependent variable in each column. Bank capital ratio is computed by first averaging across all lending banks for each firm, then averaging across firms within each industry. Firm return on assets is the ratio of net income to total assets. Industry HHI is the Herfindahl-Hirschman index of sales concentration. All regressors are lagged one period. Standard errors in parentheses.

bank capital could mean that stressed banks may tighten credit for new borrowers while simultaneously extending favorable terms to existing problem borrowers to avoid recognizing losses. It is worth noting that the coefficients on bank capital have smaller magnitudes in the asset-weighted specifications for both zombie prevalence and financial distress, suggesting that larger firms are somewhat less sensitive to bank capital constraints.

Looking at the other controls, the coefficient on firms' return on assets is negative across the board, confirming the intuition that less profitable firms are more likely to be financially distressed in the future because of their impaired ability to service their debt. Less profitable firms are also more likely to turn into zombies, which confirms that our zombie definition truly captures firm underperformance. Market concentration, as measured by the HHI, is

TABLE 11. Effect of zombie prevalence on bank capital ratios

	(1) log CAPR	(2) log CAPR	(3) log CAPR	(4) log CAPR
Zombie firms	−0.291***		−0.068	
	(0.099)		(0.110)	
Zombie firms, non-exiting		−0.290***		−0.066
		(0.100)		(0.110)
Zombie firms, exiting		−1.671**		−1.761***
		(0.829)		(0.668)
Financially distressed firms	0.280**	0.279**		
	(0.110)	(0.110)		
Firm sales growth	−0.021	−0.021	0.003	0.003
	(0.023)	(0.023)	(0.024)	(0.024)
Firm return on assets	0.555	0.555	−0.015	−0.014
	(0.594)	(0.594)	(0.533)	(0.533)
Bank fixed effects	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes
Number of banks	71	71	71	71
Number of firms	130,460	130,460	130,460	130,460

Notes: This table shows results from estimating equation (5.4). The dependent variable is the log capital ratio, defined as total equity divided by total assets. All explanatory variables are aggregated at the bank level by averaging across all of the bank's borrowers. A firm is considered financially distressed if, for the past two years, it has had above-median leverage and below-median interest coverage ratios. A firm is considered a zombie if it is financially distressed and receives an interest rate below that of AAA-rated industry peers. In columns 2 and 4, zombie firms are separated into those that exit the sample in the following period (exiting) and those that remain (non-exiting). All regressors are lagged one period. Standard errors in parentheses.

positively related to both zombie prevalence and financial distress, possibly indicating that concentrated industries may have fewer competitive pressures to eliminate weak firms.

6.2. Effect of Zombie Prevalence on Banks

Having established that poorly capitalized banks lead to zombie proliferation, we now examine the reverse question: how zombie prevalence affects bank capital ratios. Table 11 presents our findings on this feedback mechanism, which we obtain from equation (5.4).

Column 1 of Table 11 reports our key finding: an increased share of zombie firms among a bank's borrowers leads to significantly lower capital ratios in the following year. Importantly, this effect occurs while controlling for the share of financially distressed firms in the bank's

TABLE 12. Effect of zombie prevalence on bank loan loss provision ratios

	(1) log LLPR	(2) log LLPR	(3) log LLPR	(4) log LLPR
Zombie firms	−2.181*		−0.155	
	(1.319)		(1.152)	
Zombie firms, non-exiting		−2.191*		−0.160
		(1.323)		(1.155)
Zombie firms, exiting		6.050		4.765
		(3.763)		(3.083)
Financially distressed firms	3.028**	3.028**		
	(1.381)	(1.382)		
Firm sales growth	−0.541	−0.539	−0.116	−0.114
	(0.459)	(0.458)	(0.339)	(0.338)
Firm return on assets	9.490	9.490	1.980	1.979
	(6.643)	(6.644)	(5.437)	(5.437)
Bank fixed effects	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes
Number of banks	58	58	58	58
Number of firms	130,383	130,383	130,383	130,383

Notes: This table shows results from estimating equation (5.4). The dependent variable is the log of the loan loss provision ratio, defined as loan loss provisions divided by gross loans. All explanatory variables are aggregated at the bank level by averaging across all of the bank's borrowers. A firm is considered financially distressed if, for the past two years, it has had above-median leverage and below-median interest coverage ratios. A firm is considered a zombie if it is financially distressed and receives an interest rate below that of AAA-rated industry peers. In columns 2 and 4, zombie firms are separated into those that exit the sample in the following period (exiting) and those that remain (non-exiting). All regressors are lagged one period. Standard errors in parentheses.

portfolio. The coefficient on zombie prevalence therefore captures the additional impact of providing interest rate subsidies to distressed borrowers, beyond the baseline effect of lending to financially constrained firms. As for the magnitude of the effect, the coefficient on zombie firms implies that a one-percentage-point increase in the share of zombie borrowers leads to a reduction in bank capital of 3 basis points.[18] Interestingly, financial distress itself has the opposite effect on bank capital ratios. Banks with higher exposure to financially distressed borrowers actually experience better capital outcomes, suggesting that distressed firms without subsidies may trigger better risk management by banks.

[18]See also Table 27 in the Appendix, where the outcome is the actual capital ratio instead of its logarithm.

Column 2 of Table 11 provides additional insight by separating zombie firms into those that survive and those that exit in the following period. The detrimental effect of zombies on bank capitalization is primarily driven by surviving zombies rather than exiting ones. This distinction is important because it helps us avoid capturing the mechanical negative effect on bank capital that occurs when banks must write off unpaid loans from liquidated firms. While the coefficient on exiting zombies remains significantly negative, as expected from loan write-offs, the effect of non-exiting zombies is nearly identical to the aggregate zombie effect, confirming that ongoing zombie relationships are the primary source of capital deterioration.

Columns 3 and 4 of Table 11 show what happens when we remove the explicit control for financial distress. Without this control, the zombie effect largely disappears, with the exception of exiting zombies which retain their negative effect. This implies that the two components of zombie status—financial distress and interest subsidies—have opposing effects on bank outcomes and that financial distress becomes problematic for banks only when combined with forbearance. Financial distress alone may actually improve bank health by making banks more prudent, but the combination with subsidized lending hurts bank capital.

These results offer a novel insight about zombie lending practices. While keeping zombie firms afloat may allow banks to postpone recognizing losses in the short run, this strategy ultimately undermines bank capital over time. Zombie firms represent a double burden for banks: they have impaired ability to repay their loans and simultaneously receive artificially low interest rates, both of which act as drags on bank income and capital.

The opposing effects of financial distress and zombie prevalence on capital ratios highlight the importance of distinguishing between these two borrower characteristics. To explore the underlying mechanism, we examine how these factors affect banks' loan loss provisioning behavior. Table 12 presents results from estimating equation (5.4) using loan loss provision ratios as the dependent variable.

The pattern seen in column 1 helps explain the capital ratio results. Banks increase their loan loss provisions significantly in response to higher shares of financially distressed borrowers, consistent with prudent risk management practices. However, relative to this baseline response to financial distress, they provision substantially less against zombie firms.

Similar to the capital ratio results, the negative zombie effect on provisioning is driven primarily by surviving zombies (column 2), with the coefficient on non-exiting zombies being nearly identical to the aggregate effect. The effect again disappears when financial distress is not explicitly controlled for (columns 3-4), likely because the positive provisioning response to distress offsets the negative response to interest subsidies when the two are not separated.

Taking stock. Together, Tables 11 and 12 paint a consistent picture of bank behavior. Banks respond appropriately to observable financial distress through higher provisioning and better capital management. However, they appear to turn a blind eye to relationship borrowers who receive preferential treatment through subsidized rates, treating them as if they were less risky than their financial fundamentals would suggest.

These findings reveal two contrasting dynamics in the relationship between bank capital and firm distress. When banks become worse capitalized, overall financial distress in their portfolios increases, which appropriately leads banks to set aside more capital as a buffer against losses. This represents a stabilizing feedback mechanism that helps contain systemic risk.

In contrast, zombie prevalence creates a destabilizing dynamic. Poorer bank capitalization leads banks to extend subsidized credit to distressed borrowers, fostering zombie firm proliferation. The resulting increase in zombie prevalence subsequently undermines bank capital ratios even further, creating a vicious cycle that amplifies both bank fragility and economic inefficiency. This destabilizing feedback loop represents a previously unrecognized channel through which zombie lending can threaten financial stability.

Our evidence of feedback effects has important implications for zombie lending mitigation. Policy interventions to reduce zombie lending—whether through improved insolvency regimes (Becker and Ivashina, 2022; Andrews and Petroulakis, 2019), bankruptcy reforms (Kulkarni, Ritadhi, Vij, and Waldock, 2025), or direct bank inspections (Passalacqua, Angelini, Lotti, and Soggia, 2021; Bonfim, Cerqueiro, Degryse, and Ongena, 2023)—may yield even larger benefits than previously thought, as breaking the zombie-bank feedback loop could prevent the amplification of initial shocks to either bank or firm health.

6.3. Robustness

This section presents a series of robustness checks to validate our main findings. For brevity, detailed results are presented in Appendix A.2.

Adding weights to the bank equation. In a first robustness test, we replace the equal firm weights used in our baseline bank-level aggregation with debt weights when computing bank-level exposures. The rationale is that banks should be more sensitive to firms that constitute a larger share of their lending portfolio. We do not use debt weights in our baseline specification because we only observe total firm debt rather than bank-specific loan amounts, making this measure imprecise for firms borrowing from multiple banks since we attribute the firm's total debt to all of its lenders.

Despite this measurement limitation, our results remain very similar under debt weighting, as shown in Tables 14 and 15. The zombie effect on bank capital ratios persists with similar magnitude and significance, and the negative provisioning response to zombie firms remains highly significant.

Varying the zombie definition. Our main results are robust to alternative zombie definitions. We modify the definition by imposing only the high-leverage criterion as a condition for financial distress, removing the interest coverage ratio requirement. Tables 16–18 present results from our bank-to-zombie and zombie-to-bank specifications using capital ratios and loan loss provisions as dependent variables, respectively.

The coefficients remain similar in magnitude and significance to our baseline results. The main difference appears in the loan loss provision analysis, where the coefficient on financial distress remains positive but loses statistical significance. This suggests that leverage alone may not constitute a sufficient red flag for banks to increase provisioning, unlike the combination of high leverage and low interest coverage ratios used in our main specification.

Importantly, banks still experience better capital outcomes when exposed to highly leveraged firms, indicating that prudent behavior persists even when not reflected in provisioning decisions. Most crucially, banks continue to provision significantly less against zombie firms, reinforcing that the interest rate subsidy, rather than just financial distress, is the key driver of banks' change in behavior.

We also weight firm-level outcomes in the leverage-only bank specification, and results are largely unchanged (Tables 19 and 20).

No interpolation. As a further robustness check, we exclude banks with any interpolated observations and retain only those with complete raw data panels. This approach comes at the cost of a reduced sample size. Tables 21-23 present the results.

The core feedback loop between bank capital and zombie prevalence remains robust: poorly capitalized banks continue to engage in zombie lending, and zombie firms continue to undermine bank capital ratios. However, the loan loss provision results lose statistical significance, which we attribute to the substantially reduced sample size.

Consolidated accounts. We also examine our results using consolidated bank accounts instead of the unconsolidated accounts from our baseline specification. We prioritize unconsolidated accounts in our main analysis to capture maximum variation at the individual bank level, which may be dampened when aggregating across entities within banking groups. Tables 24-26 show that the core feedback mechanism between bank capital and zombie prevalence remains robust, but the loan loss provision mechanism loses statistical significance, likely due to heterogeneity in provisioning behavior across individual banks being smoothed out in consolidated accounts.

7. Conclusion

This paper investigates the feedback effects between bank health and the proliferation of zombie firms using detailed firm-bank matched data from Spain between 2005 and 2014. We develop an empirical methodology in a linear panel regression framework that addresses the inherent cross-sectional dependence between industry and bank outcomes arising from the network of firm-bank relationships.

Consistent with much of the literature on zombie lending, we find that worse-capitalized banks lead to a proliferation of zombie firms: a one-percentage-point decrease in average bank capital ratios increases the zombie share in an industry by 3.3 percentage points. Crucially, we find a previously undocumented effect in the opposite direction: zombies also worsen bank capitalization, with a one-percentage-point increase in the share of zombie borrowers reducing a bank's capital ratio by 3 basis points. We link this effect to banks' provisioning behavior: while they respond appropriately to observable financial distress through higher provisioning, they systematically under-provision against relationship borrowers receiving subsidized rates, allowing hidden losses to accumulate and ultimately undermine bank capital.

The network structure of bank-firm relationships means that we cannot assume bank and industry outcomes to be independent and forces us to account for cross-sectional dependence in regressors that standard panel methods ignore. Our empirical methodology, which exhibits stable finite sample properties even in the presence of dense networks, provides a framework for future research examining feedback effects in settings where complex networks between banks and firms pose a challenge for standard statistical inference.

While previous literature has established that zombies harm the real economy by crowding out healthy firms and depressing aggregate economic activity, their impact on the banking system itself has remained unexplored. Our paper fills this gap by showing that zombies not only emerge from weak banks but also actively hurt bank health, creating a destabilizing cycle.

This implies that the economic costs of zombie lending go beyond the well-documented real-sector inefficiencies. By uncovering the self-reinforcing dynamic between zombie prevalence and bank fragility, we show that forbearance lending also poses a threat to financial stability through the erosion of bank capital.

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Appendix

A.1. Measurement Error Analysis

A.1.1. **Measured Zombie Prevalence.** Suppose that there are two types of firms, H (healthy firm) and L (financially distressed firm). For each $i \in \{H, L\}$, R_f denotes the actual interest payment of a firm of type f and R_f^* the counterfactual interest payment of the firm f in a world without zombie lending.

Denote π_f to be the profit of the firm f. Then, the actual interest coverage ratio (ICR) and the counterfactual interest coverage ratio are defined as follows:

$$\mathrm{ICR}_f = \frac{\pi_f}{R_f} \text{ and } \quad \mathrm{ICR}_f^* = \frac{\pi_f}{R_f^*}.$$

We take ICR as a measure of the financial health of a firm.

As for (a), we define a firm f to be healthy if and only if

$$\mathrm{ICR}_f^* := \frac{\pi_f}{R_f^*} \geq \tau_U,$$

and to be financially distressed if

$$\mathrm{ICR}_f^* := \frac{\pi_f}{R_f^*} < \tau_L.$$

In our empirical study, we also consider the leverage ratio as part of the indicator of the financial distress of a firm. However, the leverage ratio does not involve the interest payment and does not suffer from the measurement error. For brevity, we omit leverage ratio in this analysis.

As for the subsidized lending, it is natural to define a firm to receive subsidized credit if and only if $r_f < r_f^*$, where r_f^* denotes the counterfactual interest rate without zombie lending and r_f the actual interest rate in the world that allows for zombie lending. Since we do not observe the actual interest rate, we use the inferred interest rates as follows:

$$r_f^* = \frac{R_f^*}{\text{Debt}_f} \text{ and } r_f = \frac{R_f}{\text{Debt}_f},$$

where Debt_f denotes the total debt of firm. We define a firm to receive a subsidized credit if

$$r_f < r_f^*.$$

Note that r_f^* is a counterfactual quantity because the interest payment R_f^* is a counterfactual interest payment. We define Z_f^* to be a zombie indicator of firm f: $Z_f^* = 1$ if the firm f is a zombie firm and $Z_f^* = 0$ otherwise. Then, our considerations so far suggest the following definition: for each firm f,

$$Z_f^* = \begin{cases} 1 & \text{if } \text{ICR}_f^* < \tau_L \text{ and } r_f < r_f^*, \\ 0 & \text{otherwise.} \end{cases}$$

We cannot construct the indicator variable Z_f^* using data, because we do not observe the counterfactual interest payment R_f^* for the financially distressed firms, due the potential presence of zombie lending. The infeasibility is not due to a particular deficiency in our data. It stems from the counterfactual nature of the zombie definition. To address this issue, we take the inferred interest rate as a proxy for R_L^* is taken to be

$$\widehat{r}_L = r_H = \frac{R_H}{\text{Debt}_H}.$$

As for the ICR, we use ICR in place of ICR* as a proxy as previously. Then, we define a feasible zombie indicator using this proxy and the redefined $\widehat{r}$: for each firm f,

$$Z_f = \begin{cases} 1 & \text{if } \text{ICR}_f \leq \tau_L \text{ and } r_f < \widehat{r}_f, \\ 0 & \text{otherwise.} \end{cases}$$

We explore how the proxy Z_f is related to the original zombie definition Z_f^*. To facilitate the comparison, we make the following assumptions.

(A-i) $R_H = R_H^*$.

(A-ii) $R_L \leq R_L^*$.

(A-iii) $r_H^* \leq r_L^*$.

Condition (A-i) says that the actual interest payment of a healthy firm is the same as the counterfactual interest payment of the same firm, because zombie lending does not affect the healthy firms. Condition (A-ii) says that the actual interest payment for the financially distressed firms can be lower than the counterfactual interest payment of the same firm, due to the possible presence of zombie lending. Condition (A-iii) says that the inferred interest rate for the healthy firm is lower than that for the financially distressed firm in a counterfactual world without zombie lending.

Then, a simple consequence of the assumptions is that Z_L is a conservative proxy for Z_L^*.

Proposition A.1. *Under the assumptions (A-i)-(A-iii), $Z_L \leq Z_L^*$.*

Proof: Note that by Condition (A-ii),

$$\text{ICR}_L \geq \text{ICR}_L^*$$

On the other hand, by Conditions (A-i) and (A-iii), we have

$$\widehat{r}_L = \frac{R_H}{\text{Debt}_H} = \frac{R_H^*}{\text{Debt}_H} = r_H^* \leq r_L^*.$$

Hence, we obtain a desired result. ■

A.1.2. **Sign Identification with Measurement Error.** Suppose that the linear panel regression model actually suffers from measurement error. In this case, we provide sufficient conditions under which the probability limits of the least squares estimators identify the sign of the true coefficients in the regression.

First, we consider the regression model for the true zombie prevalence, not the measured zombie prevalence:

$$Z^*_{i,t} = X'_{i,t-1}\alpha_1 + \delta^*_{bz}\overline{B}_{i,t-1} + u_{i,t}, \text{ and}$$
$$\log B_{\ell,t} = \overline{X}'_{\ell,t-1}\beta_1 + \delta^*_{zb}\overline{Z}^*_{\ell,t-1} + v_{\ell,t},$$

where $Z^*_{i,t}$ and $\overline{Z}^*_{\ell,t}$ denote the true zombie prevalence at the industry level and bank level respectively. We focus on the coefficients δ^*_{bz} and δ^*_{zb}.

The main challenge here is that $Z^*_{i,t}$ and $\overline{Z}^*_{\ell,t}$ are not directly observable. We observe only their proxies $Z_{i,t}$ and $\overline{Z}_{\ell,t}$. We will introduce mild conditions under which at least the sign of δ^*_{zb} and δ^*_{bz} are identified.

For a given time series, say, x_t, $t = 1, ..., T$, we introduce its *forward average*:

$$x^A_t := \frac{1}{T-t}\sum_{s=t+1}^{T} x_s.$$

Let $\mathcal{X}$ be the σ-field generated by all $X_{i,t}$'s, $f_{zb,t}$'s and $f_{bz,t}$'s, and fixed effects r_i's and r_ℓ's. We let

$$\overline{Z}_t = \frac{1}{n_B}\sum_\ell \overline{Z}_{\ell,t} \text{ and } \overline{B}_t = \frac{1}{n_I}\sum_\ell \overline{B}_{i,t},$$

where n_B denotes the number of the banks and n_I the number of the industries. We consider the following three assumptions.

Assumption A.1. The following inequalities hold for all $t = 1, ..., T$ and hold strictly for some $t = 1, ..., T-1$.

(i) $\mathrm{Var}(\overline{Z}_{\ell,t} - \overline{Z}_t \mid \mathcal{X}) \geq \mathrm{Cov}(\overline{Z}_{\ell,t} - \overline{Z}_t, (\overline{Z}_{\ell,t} - \overline{Z}_t)^A \mid \mathcal{X})$.

(ii) $\mathrm{Var}(\overline{B}_{i,t} - \overline{B}_t \mid \mathcal{X}) \geq \mathrm{Cov}(\overline{B}_{i,t} - \overline{B}_t, (\overline{B}_{i,t} - \overline{B}_t)^A \mid \mathcal{X})$.

(iii) $\mathrm{Cov}(\overline{Z}_{\ell,t} - \overline{Z}_t, \overline{Z}^*_{\ell,t} - \overline{Z}^*_t \mid \mathcal{X}) \geq \mathrm{Cov}(\overline{Z}_{\ell,t} - \overline{Z}_t, (\overline{Z}^*_{\ell,t} - \overline{Z}^*_t)^A \mid \mathcal{X})$.

Assumption A.1(i) is satisfied if the conditional covariance between $\overline{Z}_{\ell,t} - \overline{Z}_t$ and $\overline{Z}_{\ell,s} - \overline{Z}_s$ is dominated by the conditional variance of $\overline{Z}_{\ell,t} - \overline{Z}_t$. This condition is plausible because as the time displacement between the variables becomes wider, their correlation tends to become weaker. Assumption A.1(ii) has a similar flavor, now in terms of the bank health measures at the industry level. Assumption A.1(iii) says that the conditional covariance between $\overline{Z}_{\ell,t} - \overline{Z}_t$ and $\overline{Z}^*_{\ell,s} - \overline{Z}^*_s$ is weaker than their contemporaneous covariance between $\overline{Z}_{\ell,t} - \overline{Z}_t$ and $\overline{Z}^*_{\ell,t} - \overline{Z}^*_t$. This condition seems plausible when the mean-deviated proxy zombie prevalence $\overline{Z}_{\ell,t} - \overline{Z}_t$ is a reasonable proxy for the true zombie prevalence $\overline{Z}^*_{\ell,t} - \overline{Z}^*_t$, and their across-time covariation gets weaker as their times get more distant.

Assumption A.2. For all $t = 2, \ldots, T$ and all $s \geq t$, $\mathrm{Cov}(\overline{B}_{i,t-1} - \overline{B}_{t-1}, \Delta_{i,s} \mid \mathcal{X}) = 0$, where $\Delta_{i,t} = Z^*_{i,t} - Z_{i,t}$.

This condition requires that the previous period mean-deviated bank health is uncorrelated with the current or future period measurement error of zombie prevalence once conditioned on the covariates.

Assumption A.3. For all $t = 1, \ldots, T$, $i = 1, \ldots, n_I$, and $\ell = 1, \ldots, n_B$, the following statements hold.

(i) $\mathbf{E}[u_{i,t} \mid \mathcal{X}] = r_i + f_{bz,t}$.

(ii) $\mathbf{E}[v_{\ell,t} \mid \mathcal{X}] = r_\ell + f_{zb,t}$.

Assumption A.3 assumes that the covariates are strictly exogenous in both equations up to additive time effects and fixed effects.

Now, let us define

$$\delta_{zb} = \frac{\sum_{t=2}^{T} \mathrm{Cov}(\overline{Z}_{\ell,t-1} - \overline{Z}_{t-1}, (\log B_{\ell,t})^{\mathsf{H}} \mid \mathcal{X})}{\sum_{t=2}^{T} \mathrm{Cov}(\overline{Z}_{\ell,t-1} - \overline{Z}_{t-1}, \overline{Z}^{\mathsf{H}}_{\ell,t-1} \mid \mathcal{X})} \text{ and}$$

$$\delta_{bz} = \frac{\sum_{t=2}^{T} \mathrm{Cov}(\overline{B}_{i,t-1} - \overline{B}_{t-1}, Z^{\mathsf{H}}_{i,t} \mid \mathcal{X})}{\sum_{t=2}^{T} \mathrm{Cov}(\overline{B}_{i,t-1} - \overline{B}_{t-1}, \overline{B}^{\mathsf{H}}_{i,t-1} \mid \mathcal{X})}.$$

These quantities are probability limits of the method-of-moment estimators from the regression using proxies $Z_{i,t}$ and $\overline{Z}_{\ell,t}$. The following proposition shows that the sign of δ^*_{zb} is the same as that of δ_{zb}.

Proposition A.2. *(i) Suppose that Assumptions A.1(i)(iii) and A.3 hold. Then,*

$$sign(\delta^*_{zb}) = sign(\delta_{zb}).$$

(ii) Suppose that Assumptions A.1(ii), A.2 and A.3 hold. Then,

$$\delta^*_{bz} = \delta_{bz}.$$

Proof: (i) First, note that the *within-group Helmert transform* of $x_{i,t}$ is written as

$$x^{\mathsf{H}}_{i,t} = \sqrt{\frac{T-t}{T-t+1}}(x_{i,t} - \overline{x}_t - (x_{i,t} - \overline{x}_t)^A),$$

where $\overline{x}_t = \frac{1}{n}\sum_{i=1}^n x_{i,t}$. Define

$$\overline{\Delta}_{\ell,t} = \overline{Z}^*_{\ell,t} - \overline{Z}_{\ell,t}.$$

We write

$$\log B_{\ell,t} = \overline{X}'_{\ell,t-1}\beta_1 + \overline{Z}_{\ell,t-1}\delta^*_{zb} + \overline{\Delta}_{\ell,t-1}\delta^*_{zb} + \nu_{\ell,t}.$$

Hence,

$$(\log B_{\ell,t})^{\mathsf{H}} = (\overline{X}^{\mathsf{H}}_{\ell,t-1})'\beta_1 + \overline{Z}^{\mathsf{H}}_{\ell,t-1}\delta^*_{zb} + \overline{\Delta}^{\mathsf{H}}_{\ell,t-1}\delta^*_{zb} + \nu^{\mathsf{H}}_{\ell,t},$$

where the superscript "H" denotes the Helmert transform. Note that by Assumption A.3, we have $\mathbf{E}[u^{\mathsf{H}}_{i,t} \mid \mathcal{X}] = 0$ and $\mathbf{E}[v^{\mathsf{H}}_{\ell,t} \mid \mathcal{X}] = 0$. Hence,

$$\begin{aligned}\delta_{zb} &= \frac{\sum_{t=2}^{T} \mathrm{Cov}(\overline{Z}_{\ell,t-1} - \overline{Z}_{t-1}, (\log B_{\ell,t})^{\mathsf{H}} \mid \mathcal{X})}{\sum_{t=2}^{T} \mathrm{Cov}(\overline{Z}_{\ell,t-1} - \overline{Z}_{t-1}, \overline{Z}^{\mathsf{H}}_{\ell,t-1} \mid \mathcal{X})} \\ &= \delta^*_{zb} + \delta^*_{zb} \frac{\sum_{t=2}^{T} \mathrm{Cov}(\overline{Z}_{\ell,t-1} - \overline{Z}_{t-1}, \overline{\Delta}^{\mathsf{H}}_{\ell,t-1} \mid \mathcal{X})}{\sum_{t=2}^{T} \mathrm{Cov}(\overline{Z}_{\ell,t-1} - \overline{Z}_{t-1}, \overline{Z}^{\mathsf{H}}_{\ell,t-1} \mid \mathcal{X})}.\end{aligned}$$

Now, we consider

$$\begin{aligned}\mathrm{Cov}\Big(\overline{Z}_{\ell,t-1} - \overline{Z}_{t-1}, -\overline{\Delta}^{\mathsf{H}}_{\ell,t} \mid \mathcal{X}\Big) &= \mathrm{Cov}\Big(\overline{Z}_{\ell,t-1} - \overline{Z}_{t-1}, (\overline{Z}_{\ell,t-1} - \overline{Z}^*_{\ell,t-1})^{\mathsf{H}} \mid \mathcal{X}\Big) \\ &= \mathrm{Cov}\Big(\overline{Z}_{\ell,t-1} - \overline{Z}_{t-1}, \overline{Z}^{\mathsf{H}}_{\ell,t-1} \mid \mathcal{X}\Big) \\ &\quad - \mathrm{Cov}\Big(\overline{Z}_{\ell,t-1} - \overline{Z}_{t-1}, (\overline{Z}^*_{\ell,t-1})^{\mathsf{H}} \mid \mathcal{X}\Big).\end{aligned}$$

We conclude that

$$\delta_{zb} = \delta^*_{zb} \frac{\sum_{t=2}^{T} \mathrm{Cov}(\overline{Z}_{\ell,t-1} - \overline{Z}_{t-1}, (\overline{Z}^*_{\ell,t-1})^{\mathsf{H}} \mid \mathcal{X})}{\sum_{t=2}^{T} \mathrm{Cov}(\overline{Z}_{\ell,t-1} - \overline{Z}_{t-1}, \overline{Z}^{\mathsf{H}}_{\ell,t-1} \mid \mathcal{X})}. \tag{A.1}$$

The numerator is written as

$$\begin{aligned}\sum_{t=2}^{T} \sqrt{\frac{T-t}{T-t+1}} \Big(&\mathrm{Cov}(\overline{Z}_{\ell,t-1} - \overline{Z}_{t-1}, \overline{Z}^*_{\ell,t-1} - \overline{Z}^*_{t-1} \mid \mathcal{X}) \\ &- \mathrm{Cov}(\overline{Z}_{\ell,t-1} - \overline{Z}_{t-1}, (\overline{Z}^*_{\ell,t-1} - \overline{Z}^*_{t-1})^{A} \mid \mathcal{X})\Big) > 0\end{aligned}$$

by Assumption A.1(iii). Similarly, the denominator is written as

$$\begin{aligned}\sum_{t=2}^{T} \sqrt{\frac{T-t}{T-t+1}} \Big(&\mathrm{Var}(\overline{Z}_{\ell,t-1} - \overline{Z}_{t-1} \mid \mathcal{X}) \\ &- \mathrm{Cov}(\overline{Z}_{\ell,t-1} - \overline{Z}_{t-1}, (\overline{Z}_{\ell,t-1} - \overline{Z}_{t-1})^{A} \mid \mathcal{X})\Big) > 0,\end{aligned}$$

by Assumption A.1(i).

Hence, we find that

$$\frac{\sum_{t=2}^{T} \mathrm{Cov}(\overline{Z}_{\ell,t-1} - \overline{Z}_{t-1}, (\overline{Z}^{*}_{\ell,t-1})^{\mathsf{H}} \mid \mathcal{X})}{\sum_{t=2}^{T} \mathrm{Cov}(\overline{Z}_{\ell,t-1} - \overline{Z}_{t-1}, \overline{Z}^{\mathsf{H}}_{\ell,t-1} \mid \mathcal{X})} > 0.$$

In light of (A.1), this shows that

$$\mathrm{sign}(\delta_{zb}) = \mathrm{sign}(\delta^{*}_{zb}).$$

(ii) Similarly as before, we write

$$Z_{i,t} = X'_{i,t}\alpha_1 + \delta^{*}_{bz}\overline{B}_{i,t-1} - \Delta_{i,t} + u_{i,t}.$$

Hence,

$$\begin{aligned}
\delta_{bz} &= \frac{\sum_{t=2}^{T} \mathrm{Cov}(\overline{B}_{i,t-1} - \overline{B}_{t-1}, Z^{\mathsf{H}}_{i,t} \mid \mathcal{X})}{\sum_{t=2}^{T} \mathrm{Cov}(\overline{B}_{i,t-1} - \overline{B}_{t-1}, \overline{B}^{\mathsf{H}}_{i,t-1} \mid \mathcal{X})} \\
&= \delta^{*}_{bz} - \frac{\sum_{t=2}^{T} \mathrm{Cov}(\overline{B}_{i,t-1} - \overline{B}_{t-1}, \Delta^{\mathsf{H}}_{i,t} \mid \mathcal{X})}{\sum_{t=2}^{T} \mathrm{Cov}(\overline{B}_{i,t-1} - \overline{B}_{t-1}, \overline{B}^{\mathsf{H}}_{i,t-1} \mid \mathcal{X})}.
\end{aligned}$$

(Note that under Assumption A.1(ii), the denominator in the last term is positive.) Hence, the desired result follows by Assumption A.2. ■

A.2. Additional Empirical Results and Robustness

TABLE 13. Summary statistics (weighted variables)

	N	Mean	Std. Dev.	P10	P50	P90
Panel A: Industry equation						
Zombie share	5,121	0.101	0.139	0.000	0.062	0.238
Bank capital ratio	5,121	0.070	0.009	0.060	0.070	0.081
Firm sales growth	5,121	0.014	0.220	−0.185	0.025	0.186
Firm return on assets	5,121	0.028	0.049	−0.009	0.027	0.070
Industry HHI	5,121	0.091	0.147	0.005	0.039	0.225
Panel B: Bank equation						
Bank capital ratio	639	0.096	0.055	0.050	0.086	0.138
Zombie firms	639	0.163	0.184	0.000	0.120	0.409
Financially distressed firms	639	0.388	0.251	0.000	0.391	0.708
Firm sales growth	639	−0.030	0.389	−0.294	0.000	0.219
Firm return on assets	639	0.015	0.031	−0.010	0.013	0.049

Notes: This table shows summary statistics for variables in our industry-level (Panel A) and bank-level (Panel B) regressions, corresponding to equations (5.1) and (5.4), respectively. Firm-level variables are aggregated to the industry level using time-averaged asset weights in Panel A, and to the bank level using time-averaged debt weights in Panel B. A firm is considered financially distressed if, for the past two years, it has had above-median leverage and below-median interest coverage ratios. A firm is considered a zombie if it is financially distressed and receives an interest rate below that of AAA-rated industry peers. Bank capital ratio is total equity divided by total assets. Firm return on assets is net income divided by total assets. Industry HHI is the Herfindahl-Hirschman index of sales concentration.

Weighted bank exposure

TABLE 14. Effect of zombie prevalence on bank capital ratios: Weighted bank exposure

	(1) log CAPR	(2) log CAPR	(3) log CAPR	(4) log CAPR
Zombie firms		−0.208**		−0.165*
		(0.090)		(0.089)
Zombie firms, non-exiting	−0.208**		−0.165*	
	(0.090)		(0.089)	
Zombie firms, exiting	−0.202		−0.126	
	(0.176)		(0.158)	
Financially distressed firms	0.057	0.057		
	(0.072)	(0.072)		
Firm sales growth	−0.003	−0.003	0.001	0.001
	(0.013)	(0.013)	(0.013)	(0.013)
Firm return on assets	0.071	0.071	−0.017	−0.017
	(0.466)	(0.466)	(0.456)	(0.456)
Bank fixed effects	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes
Number of banks	71	71	71	71
Number of firms	130,460	130,460	130,460	130,460

Notes: This table shows results from estimating equation (5.4). The dependent variable is the log capital ratio, defined as total equity divided by total assets. All explanatory variables are aggregated at the bank level by averaging across all of the bank's borrowers using weights based on each firm's average debt size over the sample period. A firm is considered financially distressed if, for the past two years, it has had above-median leverage. A firm is considered a zombie if it is financially distressed and receives an interest rate below that of AAA-rated industry peers. In columns 2 and 4, zombie firms are separated into those that exit the sample in the following period (exiting) and those that remain (non-exiting). All regressors are lagged one period. Standard errors in parentheses.

TABLE 15. Effect of zombie share on bank loan loss provision ratios: Weighted bank exposure

	(1) log LLPR	(2) log LLPR	(3) log LLPR	(4) log LLPR
Zombie firms		-2.038^{***}		-0.670
		(0.757)		(0.785)
Zombie firms, non-exiting	-2.038^{***}		-0.671	
	(0.757)		(0.785)	
Zombie firms, exiting	-2.285^{*}		0.225	
	(1.286)		(1.207)	
Financially distressed firms	2.064^{***}	2.064^{***}		
	(0.708)	(0.708)		
Firm sales growth	-0.142	-0.142	-0.019	-0.019
	(0.168)	(0.168)	(0.129)	(0.129)
Firm return on assets	5.255	5.257	2.113	2.104
	(4.026)	(4.026)	(3.724)	(3.721)
Bank fixed effects	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes
Number of banks	58	58	58	58
Number of firms	130,383	130,383	130,383	130,383

Notes: This table shows results from estimating equation (5.4). The dependent variable is the log of the loan loss provision ratio, defined as loan loss provisions divided by gross loans. All explanatory variables are aggregated at the bank level by averaging across all of the bank's borrowers using weights based on each firm's average debt size over the sample period. A firm is considered financially distressed if, for the past two years, it has had above-median leverage. A firm is considered a zombie if it is financially distressed and receives an interest rate below that of AAA-rated industry peers. In columns 2 and 4, zombie firms are separated into those that exit the sample in the following period (exiting) and those that remain (non-exiting). All regressors are lagged one period. Standard errors in parentheses.

Leverage-only zombie definition

TABLE 16. Effect of bank capital ratios on zombie prevalence: Leverage-based zombie definition

	(1) Zombie	(2) Zombie, weighted	(3) Distressed	(4) Distressed, weighted
Bank capital ratio	-3.593^{***}	-3.316^{***}	-4.949^{***}	-4.376^{***}
	(1.349)	(1.250)	(1.848)	(1.411)
Firm sales growth	0.027	0.030^{*}	0.045^{**}	0.034^{**}
	(0.021)	(0.016)	(0.022)	(0.016)
Firm return on assets	-0.284^{*}	-0.329^{***}	-0.202	-0.210^{*}
	(0.155)	(0.113)	(0.181)	(0.114)
Industry HHI	0.381^{*}	0.168	0.616^{**}	0.185
	(0.216)	(0.271)	(0.266)	(0.296)
Industry fixed effects	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes
Number of industries	569	569	573	573
Number of firms	130,460	130,460	132,932	132,932

Notes: This table shows results from estimating equation (5.1). The dependent variable is the industry-level share of zombie firms (columns 1-2) or financially distressed firms (columns 3-4). Columns 1 and 3 use equal weights, while columns 2 and 4 use asset weights based on each firm's average asset size over the sample period. A firm is considered financially distressed if, for the past two years, it has had above-median leverage. A firm is considered a zombie if it is financially distressed and receives an interest rate below that of AAA-rated industry peers. All regressors are industry-level averages using the same weighting scheme as the dependent variable in each column. Bank capital ratio is computed by first averaging across all lending banks for each firm, then averaging across firms within each industry. Firm return on assets is the ratio of net income to total assets. Industry HHI is the Herfindahl-Hirschman index of sales concentration. All regressors are lagged one period. Standard errors in parentheses.

TABLE 17. Effect of zombie prevalence on bank capital ratios: Leverage-based zombie definition

	(1) log CAPR	(2) log CAPR	(3) log CAPR	(4) log CAPR
Zombie firms	−0.170**		0.014	
	(0.082)		(0.078)	
Zombie firms, non-exiting		−0.170**		0.014
		(0.082)		(0.078)
Zombie firms, exiting		−1.323		−1.694**
		(1.054)		(0.701)
Financially distressed firms	0.348**	0.348**		
	(0.168)	(0.168)		
Firm sales growth	−0.014	−0.014	0.000	0.000
	(0.025)	(0.025)	(0.025)	(0.025)
Firm return on assets	0.178	0.179	−0.014	−0.013
	(0.568)	(0.568)	(0.550)	(0.550)
Bank fixed effects	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes
Number of banks	71	71	71	71
Number of firms	130,460	130,460	130,460	130,460

Notes: This table shows results from estimating equation (5.4). The dependent variable is the log of the capital ratio, defined as total equity divided by total assets. All explanatory variables are aggregated at the bank level by averaging across all of the bank's borrowers. A firm is considered financially distressed if, for the past two years, it has had above-median leverage. A firm is considered a zombie if it is financially distressed and receives an interest rate below that of AAA-rated industry peers. In columns 2 and 4, zombie firms are separated into those that exit the sample in the following period (exiting) and those that remain (non-exiting). All regressors are lagged one period. Standard errors in parentheses.

TABLE 18. Effect of zombie prevalence on bank loan loss provision ratios: Leverage-based zombie definition

	(1) log LLPR	(2) log LLPR	(3) log LLPR	(4) log LLPR
Zombie firms	−2.996*		−1.839**	
	(1.766)		(0.929)	
Zombie firms, non-exiting		−2.995*		−1.838**
		(1.767)		(0.929)
Zombie firms, exiting		4.478		1.732
		(6.156)		(4.317)
Financially distressed firms	2.291	2.293		
	(2.509)	(2.511)		
Firm sales growth	−0.465	−0.463	−0.219	−0.218
	(0.489)	(0.488)	(0.323)	(0.323)
Firm return on assets	5.016	5.016	3.621	3.621
	(6.250)	(6.251)	(5.488)	(5.488)
Bank fixed effects	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes
Number of banks	58	58	58	58
Number of firms	130,383	130,383	130,383	130,383

Notes: This table shows results from estimating equation (5.4). The dependent variable is the log of the loan loss provision ratio, defined as loan loss provisions divided by gross loans. All explanatory variables are aggregated at the bank level by averaging across all of the bank's borrowers. A firm is considered financially distressed if, for the past two years, it has had above-median leverage. A firm is considered a zombie if it is financially distressed and receives an interest rate below that of AAA-rated industry peers. In columns 2 and 4, zombie firms are separated into those that exit the sample in the following period (exiting) and those that remain (non-exiting). All regressors are lagged one period. Standard errors in parentheses.

Leverage-only definition with debt weights

TABLE 19. Effect of zombie prevalence on bank capital ratios: Leverage-based zombie definition with debt-weighted bank exposure

	(1) log CAPR	(2) log CAPR	(3) log CAPR	(4) log CAPR
Zombie firms		−0.140**		−0.096
		(0.071)		(0.071)
Zombie firms, non-exiting	−0.140**		−0.096	
	(0.071)		(0.071)	
Zombie firms, exiting	−0.276		−0.218	
	(0.181)		(0.188)	
Financially distressed firms	0.090	0.090		
	(0.120)	(0.120)		
Firm sales growth	−0.006	−0.006	−0.003	−0.003
	(0.015)	(0.015)	(0.014)	(0.014)
Firm return on assets	0.162	0.163	0.131	0.132
	(0.434)	(0.434)	(0.443)	(0.443)
Bank fixed effects	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes
Number of banks	71	71	71	71
Number of firms	130,460	130,460	130,460	130,460

Notes: This table shows results from estimating equation (5.4). The dependent variable is the log of the capital ratio, defined as total equity divided by total assets. All explanatory variables are aggregated at the bank level by averaging across all of the bank's borrowers using weights based on each firm's average debt size over the sample period. A firm is considered financially distressed if, for the past two years, it has had above-median leverage. A firm is considered a zombie if it is financially distressed and receives an interest rate below that of AAA-rated industry peers. In columns 2 and 4, zombie firms are separated into those that exit the sample in the following period (exiting) and those that remain (non-exiting). All regressors are lagged one period. Standard errors in parentheses.

TABLE 20. Effect of zombie prevalence on bank loan loss provision ratios: Leverage-based zombie definition with debt-weighted bank exposure

	(1) log LLPR	(2) log LLPR	(3) log LLPR	(4) log LLPR
Zombie firms		−2.416***		−1.902***
		(0.691)		(0.587)
Zombie firms, non-exiting	−2.417***		−1.902***	
	(0.691)		(0.587)	
Zombie firms, exiting	−2.892*		−2.143	
	(1.499)		(1.334)	
Financially distressed firms	1.002	1.001		
	(0.840)	(0.840)		
Firm sales growth	−0.124	−0.124	−0.090	−0.090
	(0.126)	(0.126)	(0.123)	(0.123)
Firm return on assets	3.203	3.207	2.876	2.879
	(3.990)	(3.989)	(3.897)	(3.896)
Bank fixed effects	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes
Number of banks	58	58	58	58
Number of firms	130,383	130,383	130,383	130,383

Notes: This table shows results from estimating equation (5.4). The dependent variable is the log of the loan loss provision ratio, defined as loan loss provisions divided by gross loans. All explanatory variables are aggregated at the bank level by averaging across all of the bank's borrowers using weights based on each firm's average debt size over the sample period. A firm is considered financially distressed if, for the past two years, it has had above-median leverage. A firm is considered a zombie if it is financially distressed and receives an interest rate below that of AAA-rated industry peers. In columns 2 and 4, zombie firms are separated into those that exit the sample in the following period (exiting) and those that remain (non-exiting). All regressors are lagged one period. Standard errors in parentheses.

No interpolation

TABLE 21. Effect of bank capital ratios on zombie prevalence: No interpolation

	(1) Zombie	(2) Zombie, weighted	(3) Distressed	(4) Distressed, weighted
Bank capital ratio	-3.316^{**}	-2.681^{**}	-5.678^{***}	-3.596^{***}
	(1.322)	(1.100)	(1.963)	(1.301)
Firm sales growth	0.002	0.018	0.001	-0.002
	(0.019)	(0.014)	(0.022)	(0.017)
Firm return on assets	-0.498^{***}	-0.484^{***}	-0.744^{***}	-0.476^{***}
	(0.124)	(0.134)	(0.187)	(0.131)
Industry HHI	0.442^{**}	0.421^{*}	0.595^{**}	0.463^{*}
	(0.195)	(0.233)	(0.270)	(0.278)
Industry fixed effects	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes
Number of industries	569	569	573	573
Number of firms	130,188	130,188	132,653	132,653

Notes: This table shows results from estimating equation (5.1) using only banks with complete raw data panels, excluding any observations with interpolated values. The dependent variable is the industry-level share of zombie firms (columns 1-2) or financially distressed firms (columns 3-4). Columns 1 and 3 use equal weights, while columns 2 and 4 use asset weights based on each firm's average asset size over the sample period. A firm is considered financially distressed if, for the past two years, it has had above-median leverage and below-median interest coverage ratios. A firm is considered a zombie if it is financially distressed and receives an interest rate below that of AAA-rated industry peers. All regressors are industry-level averages using the same weighting scheme as the dependent variable in each column. Bank capital ratio is computed by first averaging across all lending banks for each firm, then averaging across firms within each industry. Firm return on assets is the ratio of net income to total assets. Industry HHI is the Herfindahl-Hirschman index of sales concentration. All regressors are lagged one period. Standard errors in parentheses.

TABLE 22. Effect of zombie prevalence on bank capital ratios: No interpolation

	(1) log CAPR	(2) log CAPR	(3) log CAPR	(4) log CAPR
Zombie firms		-0.240^{**}		-0.008
		(0.096)		(0.097)
Zombie firms, non-exiting	-0.239^{**}		-0.007	
	(0.096)		(0.098)	
Zombie firms, exiting	-2.524^{***}		-2.412^{***}	
	(0.481)		(0.457)	
Financially distressed firms	0.291^{**}	0.291^{**}		
	(0.118)	(0.118)		
Firm sales growth	-0.016	-0.015	0.003	0.003
	(0.026)	(0.026)	(0.028)	(0.028)
Firm return on assets	1.005^{*}	1.003^{*}	0.367	0.365
	(0.597)	(0.597)	(0.571)	(0.571)
Bank fixed effects	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes
Number of banks	60	60	60	60
Number of firms	130,188	130,188	130,188	130,188

Notes: This table shows results from estimating equation (5.4) using only banks with complete raw data panels, excluding any observations with interpolated values. The dependent variable is the log of the capital ratio, defined as total equity divided by total assets. All explanatory variables are aggregated at the bank level by averaging across all of the bank's borrowers. A firm is considered financially distressed if, for the past two years, it has had above-median leverage and below-median interest coverage ratios. A firm is considered a zombie if it is financially distressed and receives an interest rate below that of AAA-rated industry peers. In columns 2 and 4, zombie firms are separated into those that exit the sample in the following period (exiting) and those that remain (non-exiting). All regressors are lagged one period. Standard errors in parentheses.

TABLE 23. Effect of zombie prevalence on bank loan loss provision ratios: No interpolation

	(1) log LLPR	(2) log LLPR	(3) log LLPR	(4) log LLPR
Zombie firms		−4.100 (2.678)		−0.672 (2.062)
Zombie firms, non-exiting	−4.111 (2.685)		−0.678 (2.069)	
Zombie firms, exiting	1.774 (3.688)		3.184 (3.813)	
Financially distressed firms	4.710** (2.395)	4.707** (2.395)		
Firm sales growth	−1.768 (1.397)	−1.771 (1.398)	−0.432 (0.868)	−0.435 (0.869)
Firm return on assets	13.483 (8.468)	13.500 (8.471)	5.378 (6.458)	5.392 (6.454)
Bank fixed effects	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes
Number of banks	44	44	44	44
Number of firms	123,451	123,451	123,451	123,451

Notes: This table shows results from estimating equation (5.4) using only banks with complete raw data panels, excluding any observations with interpolated values. The dependent variable is the log of the loan loss provision ratio, defined as loan loss provisions divided by gross loans. All explanatory variables are aggregated at the bank level by averaging across all of the bank's borrowers. A firm is considered financially distressed if, for the past two years, it has had above-median leverage and below-median interest coverage ratios. A firm is considered a zombie if it is financially distressed and receives an interest rate below that of AAA-rated industry peers. In columns 2 and 4, zombie firms are separated into those that exit the sample in the following period (exiting) and those that remain (non-exiting). All regressors are lagged one period. Standard errors in parentheses.

Consolidated accounts

TABLE 24. Effect of bank capital ratios on zombie prevalence: Consolidated accounts

	(1) Zombie	(2) Zombie, weighted	(3) Distressed	(4) Distressed, weighted
Bank capital ratio	−1.960*	−1.109	−2.102*	−0.643
	(1.134)	(0.927)	(1.264)	(1.184)
Firm sales growth	0.001	0.017	0.003	−0.002
	(0.018)	(0.014)	(0.021)	(0.017)
Firm return on assets	−0.538***	−0.495***	−0.808***	−0.477***
	(0.132)	(0.136)	(0.199)	(0.133)
Industry HHI	0.445**	0.418*	0.589**	0.462
	(0.201)	(0.235)	(0.284)	(0.282)
Industry fixed effects	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes
Number of industries	569	569	573	573
Number of firms	130,460	130,460	132,932	132,932

Notes: This table shows results from estimating equation (5.1) using consolidated bank accounts instead of unconsolidated accounts. The dependent variable is the industry-level share of zombie firms (columns 1-2) or financially distressed firms (columns 3-4). Columns 1 and 3 use equal weights, while columns 2 and 4 use asset weights based on each firm's average asset size over the sample period. A firm is considered financially distressed if, for the past two years, it has had above-median leverage and below-median interest coverage ratios. A firm is considered a zombie if it is financially distressed and receives an interest rate below that of AAA-rated industry peers. All regressors are industry-level averages using the same weighting scheme as the dependent variable in each column. Bank capital ratio is computed by first averaging across all lending banks for each firm, then averaging across firms within each industry. Firm return on assets is the ratio of net income to total assets. Industry HHI is the Herfindahl-Hirschman index of sales concentration. All regressors are lagged one period. Standard errors in parentheses.

TABLE 25. Effect of zombie prevalence on bank capital ratios: Consolidated accounts

	(1) log CAPR	(2) log CAPR	(3) log CAPR	(4) log CAPR
Zombie firms		−0.295***		−0.074
		(0.100)		(0.109)
Zombie firms, non-exiting	−0.294***		−0.073	
	(0.100)		(0.110)	
Zombie firms, exiting	−2.064***		−2.153***	
	(0.581)		(0.517)	
Financially distressed firms	0.276**	0.277**		
	(0.109)	(0.109)		
Firm sales growth	−0.021	−0.020	0.003	0.003
	(0.023)	(0.023)	(0.024)	(0.024)
Firm return on assets	0.570	0.570	0.006	0.006
	(0.588)	(0.588)	(0.528)	(0.528)
Bank fixed effects	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes
Number of banks	71	71	71	71
Number of firms	130,460	130,460	130,460	130,460

Notes: This table shows results from estimating equation (5.4) using consolidated bank accounts instead of unconsolidated accounts. The dependent variable is the log of the capital ratio, defined as total equity divided by total assets. All explanatory variables are aggregated at the bank level by averaging across all of the bank's borrowers. A firm is considered financially distressed if, for the past two years, it has had above-median leverage and below-median interest coverage ratios. A firm is considered a zombie if it is financially distressed and receives an interest rate below that of AAA-rated industry peers. In columns 2 and 4, zombie firms are separated into those that exit the sample in the following period (exiting) and those that remain (non-exiting). All regressors are lagged one period. Standard errors in parentheses.

TABLE 26. Effect of zombie prevalence on bank loan loss provision ratios: Consolidated accounts

	(1) log LLPR	(2) log LLPR	(3) log LLPR	(4) log LLPR
Zombie firms		−2.019		−0.049
		(1.309)		(1.146)
Zombie firms, non-exiting	−2.027		−0.054	
	(1.313)		(1.149)	
Zombie firms, exiting	5.236*		3.986	
	(3.090)		(3.369)	
Financially distressed firms	2.943**	2.943**		
	(1.374)	(1.373)		
Firm sales growth	−0.514	−0.516	−0.101	−0.102
	(0.447)	(0.447)	(0.331)	(0.331)
Firm return on assets	9.686	9.686	2.386	2.387
	(6.576)	(6.576)	(5.397)	(5.397)
Bank fixed effects	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes
Number of banks	58	58	58	58
Number of firms	130,383	130,383	130,383	130,383

Notes: This table shows results from estimating equation (5.4) using consolidated bank accounts instead of unconsolidated accounts. The dependent variable is the log of the loan loss provision ratio, defined as loan loss provisions divided by gross loans. All explanatory variables are aggregated at the bank level by averaging across all of the bank's borrowers. A firm is considered financially distressed if, for the past two years, it has had above-median leverage and below-median interest coverage ratios. A firm is considered a zombie if it is financially distressed and receives an interest rate below that of AAA-rated industry peers. In columns 2 and 4, zombie firms are separated into those that exit the sample in the following period (exiting) and those that remain (non-exiting). All regressors are lagged one period. Standard errors in parentheses.

No log transformation

TABLE 27. Effect of zombie prevalence on bank capital ratios

	(1) CAPR	(2) CAPR	(3) CAPR	(4) CAPR
Zombie firms		−0.030***		−0.015
		(0.009)		(0.011)
Zombie firms, non-exiting	−0.030***		−0.015	
	(0.009)		(0.011)	
Zombie firms, exiting	−0.162**		−0.168***	
	(0.069)		(0.057)	
Financially distressed firms	0.019	0.019		
	(0.013)	(0.013)		
Firm sales growth	0.002	0.002	0.004	0.004
	(0.005)	(0.005)	(0.005)	(0.005)
Firm return on assets	0.051	0.051	0.011	0.011
	(0.073)	(0.073)	(0.071)	(0.071)
Bank fixed effects	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes
Number of banks	71	71	71	71
Number of firms	130,460	130,460	130,460	130,460

Notes: This table shows results from estimating equation (5.4). The dependent variable is the capital ratio, defined as total equity divided by total assets.All explanatory variables are aggregated at the bank level by averaging across all of the bank's borrowers. A firm is considered financially distressed if, for the past two years, it has had above-median leverage and below-median interest coverage ratios. A firm is considered a zombie if it is financially distressed and receives an interest rate below that of AAA-rated industry peers. In columns 2 and 4, zombie firms are separated into those that exit the sample in the following period (exiting) and those that remain (non-exiting). All regressors are lagged one period. Standard errors in parentheses.

A.3. Asymptotic Inference

A.3.1. **Estimation and Inference.** Let us explain the estimation procedure. We first obtain consistent estimators of γ_{bz} and γ_{zb}. Then, using these estimators, we form an optimal weighting matrix to construct the main estimators of γ_{bz} and γ_{zb}. Define

$$\hat{M}_w = \frac{1}{n_B(T-1)} \sum_{t=2}^{T} \sum_{\ell \in N_B} (W_{\ell,t-1} - \overline{W}_{t-1}) W_{\ell,t-1}^{\mathsf{H}\prime} \text{ and}$$
$$\hat{M}_{wb} = \frac{1}{n_B(T-1)} \sum_{t=2}^{T} \sum_{\ell \in N_B} (W_{\ell,t-1} - \overline{W}_{t-1}) (\log B_{\ell,t})^{\mathsf{H}},$$

where $\overline{W}_{t-1} = \frac{1}{n_B} \sum_{\ell \in N_B} W_{\ell,t-1}$. We also define

$$\tilde{\gamma}_{zb} = \left(\hat{M}_w' \hat{M}_w\right)^{-1} \hat{M}_w' \hat{M}_{wb}.$$

Similarly, we let $\tilde{\gamma}_{bz} = \left(\hat{M}_v' \hat{M}_v\right)^{-1} \hat{M}_v' \hat{M}_{vz}$, where we use $V_{i,t-1}$ instead of $W_{\ell,t-1}$ and $Z_{i,t}$ instead of $\log B_{\ell,t}$, that is,

$$\hat{M}_v = \frac{1}{n_I(T-1)} \sum_{t=2}^{T} \sum_{i \in N_I} (V_{i,t-1} - \overline{V}_{t-1}) V_{i,t-1}^{\mathsf{H}\prime} \text{ and}$$
$$\hat{M}_{vz} = \frac{1}{n_I(T-1)} \sum_{t=2}^{T} \sum_{i \in N_I} (V_{i,t-1} - \overline{V}_{t-1}) Z_{i,t-1}^{\mathsf{H}},$$

with $\overline{V}_{t-1} = \frac{1}{n_I} \sum_{i \in N_I} V_{i,t-1}$. This gives us the first step estimators $\tilde{\gamma}_{zb}$ and $\tilde{\gamma}_{bz}$.

Now we explain asymptotic inference. First, we construct

$$\hat{\psi}_{zb,\ell} = \frac{1}{T-1} \sum_{t=2}^{T} (W_{\ell,t-1} - \overline{W}_{t-1}) \hat{v}_{\ell,t}^{\mathsf{H}}, \text{ and } \hat{\psi}_{bz,i} = \frac{1}{T-1} \sum_{t=2}^{T} (V_{i,t-1} - \overline{V}_{t-1}) \hat{u}_{i,t}^{\mathsf{H}},$$

where $\hat{v}_{\ell,t}^{\mathsf{H}} = (\log B_{\ell,t})^{\mathsf{H}} - W_{\ell,t-1}^{\mathsf{H}\prime} \tilde{\gamma}_{zb}$ and $\hat{u}_{i,t}^{\mathsf{H}} = Z_{i,t}^{\mathsf{H}} - V_{i,t-1}^{\mathsf{H}\prime} \tilde{\gamma}_{bz}$. Define

$$\hat{\Omega}_{zb} = \frac{1}{n_B} \sum_{\ell \in N_B} \hat{\psi}_{zb,\ell} \hat{\psi}_{zb,\ell}' \text{ and } \hat{\Omega}_{bz} = \frac{1}{n_I} \sum_{i \in N_I} \hat{\psi}_{bz,i} \hat{\psi}_{bz,i}'.$$

Using this, we construct second step estimators as follows:

$$\hat{\gamma}_{zb} = \left(\hat{M}_w'\hat{\Omega}_{zb}^{-1}\hat{M}_w\right)^{-1}\hat{M}_w'\hat{\Omega}_{zb}^{-1}\hat{M}_{wb} \text{ and}$$
$$\hat{\gamma}_{bz} = \left(\hat{M}_v'\hat{\Omega}_{bz}^{-1}\hat{M}_v\right)^{-1}\hat{M}_v'\hat{\Omega}_{bz}^{-1}\hat{M}_{vz}.$$

Below, we show that under the regularity assumptions,

$$\sqrt{n_B}(\hat{\gamma}_{zb} - \gamma_{zb}) \to_d N(0, \Sigma_{zb}), \text{ and}$$
$$\sqrt{n_I}(\hat{\gamma}_{bz} - \gamma_{bz}) \to_d N(0, \Sigma_{bz}),$$

where $\Sigma_{zb} = \left(M_w'\Omega_{zb}^{-1}M_w\right)^{-1}$ and $\Sigma_{bz} = \left(M_v'\Omega_{bz}^{-1}M_v\right)^{-1}$ and M_w, M_v, Ω_{zb} and Ω_{bz} are probability limits of $\hat{M}_w$, $\hat{M}_v$, $\hat{\Omega}_{zb}$ and $\hat{\Omega}_{bz}$. Thus, in order to implement asymptotic inference, we estimate these asymptotic variances Σ_{zb} and Σ_{bz} as

$$\hat{\Sigma}_{zb} = \left(\hat{M}_w'\hat{\Omega}_{zb}^{-1}\hat{M}_w\right)^{-1} \text{ and } \hat{\Sigma}_{bz} = \left(\hat{M}_v'\hat{\Omega}_{bz}^{-1}\hat{M}_v\right)^{-1}.$$

Using these estimators, we construct standard errors in a standard manner.

A.3.2. **Asymptotic Theory.** For brevity, we focus on the asymptotic normality of $\sqrt{n_B}(\hat{\gamma}_{zb} - \gamma_{zb})$ only. That of $\sqrt{n_I}(\hat{\gamma}_{bz} - \gamma_{bz})$ can be dealt with similarly. For each time $t \geq 1$, we define the σ-field

$$\mathcal{F}_{t-1} = \sigma(\boldsymbol{W}_{t-1}, \boldsymbol{f}_t, \boldsymbol{r}),$$

where $\boldsymbol{W}_{t-1} = (W_s)_{s=1}^{t-1}$, and $\boldsymbol{f}_t = (f_{bz,s}, f_{zb,s})_{s=1}^{t}$, with $W_{t-1} = (W_{\ell,t-1})_{\ell=1}^{n_B}$ and $\boldsymbol{r} = (r_\ell)_{\ell=1}^{n_B}$ is the collection of all fixed effects. We set $\mathcal{F}_0 = \sigma(\boldsymbol{f}_1, \boldsymbol{r})$.

We denote the conditional expectation, variance, and covariance given $\mathcal{F}_{t-1}$ by $\mathbf{E}_{t-1}$, Var_{t-1}, and Cov_{t-1}. The first set of assumptions is concerned with the error terms $\eta_{\ell,t}$.

Assumption A.4. For each $t = 2, ..., T$, the following statements hold.

(i) $\eta_{\ell,t}$'s are conditionally independent across $\ell \in N_B$ given $\mathcal{F}_{t-1}$.

(ii) $\mathbf{E}_{t-1}[\eta_{\ell,t}] = 0$ for all $\ell \in N_B$.

(iii) There exists $C > 0$ such that for all $n_B \geq 1$, $\max_{\ell \in N_B} \mathbf{E}_{t-1}[\eta_{\ell,t}^4] \leq C$.

(iv) $\sigma_{n,t}^2 := \mathbf{E}_{t-1}[\eta_{\ell,t}^2]$ is identical across $\ell \in N_B$, and as $n_B \to \infty$,

$$\sigma_{n,t}^2 - \sigma_t^2 \to_P 0,$$

for some random variable $\sigma_t^2 > 0$ which is $\mathcal{F}_0$-measurable.

Assumption A.4(i) says that once we condition on $\mathcal{F}_{t-1}$, $\eta_{\ell,t}$'s do not exhibit any cross-sectional dependence. Hence the errors can still be cross-sectionally correlated through the variables contained in $\mathcal{F}_{t-1}$. Assumption A.4(iii) requires a conditional fourth moment condition for the errors, uniformly over $\ell \in N_B$. Assumption A.4(iv) allows for conditional heteroskedasticity for $\eta_{\ell,t}$, only if the heteroskedasticity arises through the conditioning variables in $\mathcal{F}_{t-1}$.

The assumption below is a condition that requires the networks between the banks and the firms to be sparse enough.

Assumption A.5. For any $k, m = 1, \ldots, d_W$ and $s, s', s'', t, t', t'' = 2, \ldots, T$, with $h, h', h'' = 2, \ldots, T$ such that $h < \min\{s, t\}$, $h' < \min\{s, s', t, t'\}$ and $h'' < \min\{s, s', s'', t, t', t''\}$, we have

$$\begin{aligned}
&\frac{1}{n_B} \sum_{\ell \in N_B} \sum_{\ell' \in N_B} \mathrm{Cov}_h \left(W_{\ell,t,k}, W_{\ell',s,m} \right) = O_P(1), \\
&\frac{1}{n_B^2} \sum_{\ell, \ell' \in N_B} \sum_{\tilde{\ell}, \tilde{\ell}' \in N_B} \mathrm{Cov}_{h'} \left(W_{\ell,t,k} W_{\tilde{\ell},t',k}, W_{\ell',s,m} W_{\tilde{\ell}',s',m} \right) = O_P(1), \text{ and} \\
&\frac{1}{n_B^2} \sum_{\ell, \ell' \in N_B} \sum_{\tilde{\ell}, \tilde{\ell}' \in N_B} \mathrm{Cov}_{h''} \left(W_{\ell,t,k}^2 W_{\tilde{\ell},t',k} W_{\tilde{\ell},t'',k}, W_{\ell',s,m}^2 W_{\tilde{\ell}',s',m} W_{\tilde{\ell}',s'',m} \right) = O_P(1), \text{ as } n_B \to \infty,
\end{aligned}$$

where $W_{\ell,t,j}$ denotes the j-th entry of $W_{\ell,t}$, and d_W denotes the dimension of $W_{\ell,t}$.

Note that $W_{\ell,t}$ involves the bank-level zombie prevalence measures $\overline{Z}_{\ell,t}$. As there are firms that are connected with multiple banks, $W_{\ell,t}$ naturally exhibits cross-sectional dependence across the banks. While Assumption A.5 allows for such presence of cross-sectional dependence, it restricts that for each t, the cross-sectional dependence is not too extensive among the banks, once conditioned on $\mathcal{F}_{t-1}$.

The fourth assumption is concerned with the limit of the components in the conditional variance of the estimator $\hat{\gamma}_{zb}$. Define

$$\tilde{W}_{\ell,s} = \sum_{t=2}^{s} h_{s,t}(W_{\ell,t} - \overline{W}_t) \text{ and } \tilde{W}^*_{\ell,s} = \sum_{t=2}^{s} h_{s,t}(W_{\ell,t} - \mathbf{E}_{t-1}[\overline{W}_t]),$$

and

$$M_{n,w} = \sum_{s=2}^{T} \frac{1}{n_B} \sum_{\ell \in N_B} \mathbf{E}_{s-2}\left[\tilde{W}^*_{\ell,s-1} W'_{\ell,s}\right], \text{ and} \tag{A.2}$$
$$\Omega_{n,s} = \frac{1}{n_B} \sum_{\ell \in N_B} \mathbf{E}_{s-2}\left[\sigma^2_{n,s} \tilde{W}^*_{\ell,s-1} \tilde{W}^{*\prime}_{\ell,s-1}\right], \text{ for each } s = 2, \ldots, T,$$

where we recall $\sigma^2_{n,s} = \mathbf{E}_{s-1}[\eta^2_{\ell,s}]$. As for $M_{n,w}$ and $\Omega_{n,s}$, we make the following assumption.

Assumption A.6. For $s = 1, \ldots, T$, the following holds.

(i) There exist $c > 0$ and $n_0 \geq 1$ such that for all $n_B \geq n_0$,

$$\lambda_{\min}\left(\frac{1}{n_B} \sum_{\ell \in N_B} \mathbf{E}_{s-2}\left[\tilde{W}^*_{\ell,s-1} \tilde{W}^{*\prime}_{\ell,s-1}\right]\right) > c,$$

where $\lambda_{\min}(A)$ for any symmetric matrix A denotes the minimum eigenvalue of A.

(ii) There exist $\mathcal{F}_0$-measurable random matrices M_w and Ω_s such that as $n_B \to \infty$,

$$M_{n,w} - M_w \to_P 0 \text{ and } \Omega_{n,s} - \Omega_s \to_P 0,$$

where $M'_w M_w$ and Ω_s are nonsingular.

Assumption A.6 requires that the "effective instrumental variables", $\tilde{W}^*_{\ell,s-1}$, are not redundant for large enough sample size. While the convergence assumption (ii) may be replaced by some lower level conditions, it appears that given the heterogeneity of conditional distributions, such convergence is necessary for the asymptotic validity of the inference procedure. Such an assumption has often been used in the literature in developing asymptotic theory with heterogeneously distributed random variables. (For example, see Assumption EX of Kuersteiner and Prucha (2020).)

Lastly, we introduce conditions that control the conditional moments of the data.

Assumption A.7. There exist $C > 0$ and $n_0 \geq 1$ such that for all $n_B \geq n_0$,

$$\sum_{s=2}^{T} \frac{1}{n_B} \sum_{\ell \in N_B} \mathbf{E}_{s-2}\left[\|\tilde{W}^{*}_{\ell,s-1}\|^4 \right] \leq C, \text{ and } \sum_{s=2}^{T} \frac{1}{n_B} \sum_{\ell \in N_B} \mathbf{E}_{s-1}\left[\|W_{\ell,s}\|^4 \right] \leq C.$$

Under these assumptions, we can obtain the asymptotic normality for the estimator $\hat{\gamma}_{zb}$ as follows:

Theorem A.1. *Suppose that Assumptions A.4-A.7 hold. Then, as $n_B \to \infty$,*

$$\sqrt{n_B}\hat{\Sigma}_{zb}^{-1/2}(\hat{\gamma}_{zb} - \gamma_{zb}) \to_d N(0, I),$$

where I denotes the identity matrix.

Supplemental Note to "Zombie Lending and Bank Health: Exploring Feedback Effects"

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In this supplemental note, we provide the proof of Theorem A.1. Throughout the supplemental note, we assume that the assumptions in the theorem hold. Recall the definitions in the appendix of the paper:

$$\tilde{W}_{\ell,s} = \sum_{t=2}^{s} h_{s,t}(W_{\ell,t} - \overline{W}_t) \text{ and } \tilde{W}^*_{\ell,s} = \sum_{t=2}^{s} h_{s,t}(W_{\ell,t} - \mathbf{E}_{t-1}[\overline{W}_t]),$$

and

$$\text{(B.3)} \qquad M_{n,w} = \sum_{s=2}^{T} \frac{1}{n_B} \sum_{\ell \in N_B} \mathbf{E}_{s-2}\left[\tilde{W}^*_{\ell,s-1} W'_{\ell,s}\right], \text{ and}$$

$$\Omega_{n,s} = \frac{1}{n_B} \sum_{\ell \in N_B} \mathbf{E}_{s-2}\left[\sigma^2_{n,s} \tilde{W}^*_{\ell,s-1} \tilde{W}^{*\prime}_{\ell,s-1}\right], \text{ for each } s = 2, \dots, T,$$

where $\sigma^2_{n,s} = \mathbf{E}[\eta^2_{\ell,s} \mid \mathcal{F}_{s-1}]$. We define

$$\text{(B.4)} \qquad \xi_s = \frac{1}{\sqrt{n_B}} \sum_{\ell \in N_B} \tilde{W}^*_{\ell,s-1} \eta_{\ell,s}.$$

Lemma B.1. *As $n_B \to \infty$, $\sqrt{n_B}\hat{U}_{zb} = \sum_{s=2}^{T} \xi_s + o_P(1)$.*

Proof: Define

$$\hat{U}_{zb,t} = \sum_{s=t}^{T} h_{s,t} \frac{1}{n_B} \sum_{\ell \in N_B} (W_{\ell,t} - \overline{W}_t)(v_{\ell,s} - \overline{v}_s).$$

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We write

$$\text{(B.5)} \qquad \sqrt{n_B}\hat{U}_{zb} = \sqrt{n_B}\sum_{t=2}^{T}\hat{U}_{zb,t} = \frac{1}{\sqrt{n_B}}\sum_{t=2}^{T}\sum_{s=t}^{T}h_{s,t}\sum_{\ell\in N_B}(W_{\ell,t}-\overline{W}_t)(v_{\ell,s}-\overline{v}_s)$$
$$= \sum_{t=2}^{T}\frac{1}{\sqrt{n_B}}\sum_{\ell\in N_B}(W_{\ell,t}-\overline{W}_t)\sum_{s=t}^{T}h_{s,t}(\eta_{\ell,s}+r_\ell-(\overline{\eta}_s+\overline{r})),$$

where

$$\overline{\eta}_s = \frac{1}{n_B}\sum_{\ell\in N_B}\eta_{\ell,s}, \text{ and } \overline{r} = \frac{1}{n_B}\sum_{\ell\in N_B}r_\ell.$$

The last term on the right-hand side of (B.5) is equal to

$$\sum_{t=2}^{T}\frac{1}{\sqrt{n_B}}\sum_{\ell\in N_B}(W_{\ell,t}-\overline{W}_t)\sum_{s=t}^{T}h_{s,t}(\eta_{\ell,s}+r_\ell),$$

due to the mean deviation $W_{\ell,t}-\overline{W}_t$. By the definition of $h_{s,t}$, we have $\sum_{s=t}^{T}h_{s,t}=0$, and hence the last sum is equal to

$$\frac{1}{\sqrt{n_B}}\sum_{\ell\in N_B}\sum_{t=2}^{T}(W_{\ell,t}-\overline{W}_t)\sum_{s=t}^{T}h_{s,t}\eta_{\ell,s} = \sum_{s=2}^{T}\frac{1}{\sqrt{n_B}}\sum_{\ell\in N_B}\sum_{t=2}^{s}h_{s,t}(W_{\ell,t}-\overline{W}_t)\eta_{\ell,s}$$
$$= \sum_{s=2}^{T}\xi_s + A_n,$$

where

$$A_n = \sum_{s=2}^{T}\frac{1}{\sqrt{n_B}}\sum_{\ell\in N_B}\sum_{t=2}^{s}h_{s,t}(\mathbf{E}_{t-1}[\overline{W}_t]-\overline{W}_t)\eta_{\ell,s}.$$

Note that

$$A_n = \sum_{t=2}^{T}(\mathbf{E}_{t-1}[\overline{W}_t]-\overline{W}_t)\sum_{s=t}^{T}h_{s,t}\frac{1}{\sqrt{n_B}}\sum_{\ell\in N_B}\eta_{\ell,s} = o_P(1),$$

because for each $t=2,\dots,T$,

$$\frac{1}{\sqrt{n_B}}\sum_{\ell\in N_B}\eta_{\ell,s} = O_P(1) \text{ and } \mathbf{E}_{t-1}[\overline{W}_t]-\overline{W}_t = o_P(1),$$

by Assumptions A.4 and A.5. Thus, the desired result follows. ■

Lemma B.2. *For each $s = 2, ..., T$, as $n_B \to \infty$,*

$$\frac{1}{n_B}\sum_{\ell\in N_B}\left(\tilde{W}_{\ell,s-1}W'_{\ell,s} - \mathbf{E}_{s-2}\left[\tilde{W}^*_{\ell,s-1}W'_{\ell,s}\right]\right) \to_P 0.$$

Proof: We first write

$$\text{(B.6)}\qquad \begin{aligned}\frac{1}{n_B}\sum_{\ell\in N_B}\left(\tilde{W}_{\ell,s-1} - \tilde{W}^*_{\ell,s-1}\right)W'_{\ell,s} &= \sum_{t=2}^{s-1}\frac{1}{n_B}\sum_{\ell\in N_B}h_{s,t}(\mathbf{E}_{t-1}[\overline{W}_t] - \overline{W}_t)W'_{\ell,s}\\ &= \sum_{t=2}^{s-1}h_{s,t}(\mathbf{E}_{t-1}[\overline{W}_t] - \overline{W}_t)\frac{1}{n_B}\sum_{\ell\in N_B}W'_{\ell,s} = o_P(1),\end{aligned}$$

because for each $s, t = 2, ..., T$,

$$\frac{1}{n_B}\sum_{\ell\in N_B}W'_{\ell,s} = O_P(1) \text{ and } \mathbf{E}_{t-1}[\overline{W}_t] - \overline{W}_t = o_P(1),$$

by Assumption A.5.

Now, note that

$$\mathbf{E}_{s-2}\left[\tilde{W}^*_{\ell,s-1}W'_{\ell,s}\right] = \sum_{t=2}^{s}h_{s,t}\mathbf{E}_{s-2}\left[\left(W_{\ell,t} - \mathbf{E}_{t-1}[\overline{W}_t]\right)W'_{\ell,s}\right].$$

Hence,

$$\begin{aligned}&\frac{1}{n_B}\sum_{\ell\in N_B}\left(\tilde{W}^*_{\ell,s-1}W'_{\ell,s} - \mathbf{E}_{s-2}\left[\tilde{W}^*_{\ell,s-1}W'_{\ell,s}\right]\right)\\ &= \sum_{t=2}^{s}h_{s,t}\frac{1}{n_B}\sum_{\ell\in N_B}\left((W_{\ell,t} - \mathbf{E}_{t-1}[\overline{W}_t])W'_{\ell,s} - \mathbf{E}_{s-2}\left[(W_{\ell,t} - \mathbf{E}_{t-1}[\overline{W}_t])W'_{\ell,s}\right]\right)\\ &= \sum_{t=2}^{s}h_{s,t}A_{1n,s,t} + \sum_{t=2}^{s}h_{s,t}A_{2n,s,t},\end{aligned}$$

where

$$A_{1n,s,t} = \frac{1}{n_B} \sum_{\ell \in N_B} \left(W_{\ell,t} W'_{\ell,s} - \mathbf{E}_{s-2} \left[W_{\ell,t} W'_{\ell,s} \right] \right) \text{ and}$$

$$A_{2n,s,t} = \frac{1}{n_B} \sum_{\ell \in N_B} \left(\mathbf{E}_{t-1}[\overline{W}_t] W'_{\ell,s} - \mathbf{E}_{s-2} \left[\mathbf{E}_{t-1}[\overline{W}_t] W'_{\ell,s} \right] \right).$$

By Assumption A.5, it is not hard to see that $A_{1n,s,t} = o_P(1)$. As for $A_{2n,s,t}$, we write its (j,m)-th entry as

$$\text{(B.7)} \quad \frac{1}{n_B^2} \sum_{\ell \in N_B} \sum_{\ell' \in N_B} \left(\mathbf{E}_{t-1}[W_{\ell',t,j}] W_{\ell,s,m} - \mathbf{E}_{s-2} \left[\mathbf{E}_{t-1}[W_{\ell',t,j}] W_{\ell,s,m} \right] \right)$$
$$= \frac{1}{n_B^2} \sum_{\ell \in N_B} \sum_{\ell' \in N_B} \left(W_{\ell',t,j} W_{\ell,s,m} - \mathbf{E}_{s-2} \left[W_{\ell',t,j} W_{\ell,s,m} \right] \right)$$
$$- \frac{1}{n_B^2} \sum_{\ell \in N_B} \sum_{\ell' \in N_B} \left((W_{\ell',t,j} - \mathbf{E}_{t-1}[W_{\ell',t,j}]) W_{\ell,s,m} - \mathbf{E}_{s-2} \left[(W_{\ell',t,j} - \mathbf{E}_{t-1}[W_{\ell',t,j}]) W_{\ell,s,m} \right] \right).$$

The first term on the right hand side is written as

$$\frac{1}{n_B^2} \sum_{\ell \in N_B} \sum_{\ell' \in N_B} \left(W_{\ell',t,j} W_{\ell,s,m} - \mathbf{E}_{s-2} \left[W_{\ell',t,j} W_{\ell,s,m} \right] \right)$$
$$= \frac{1}{n_B^2} \sum_{\ell \in N_B} \sum_{\ell' \in N_B} \left(W_{\ell',t,j} W_{\ell,s,m} - \mathbf{E}_{s-1} \left[W_{\ell',t,j} W_{\ell,s,m} \right] \right)$$
$$+ \frac{1}{n_B^2} \sum_{\ell \in N_B} \sum_{\ell' \in N_B} \mathbf{E}_{s-1} \left[W_{\ell',t,j} W_{\ell,s,m} - \mathbf{E}_{s-2} \left[W_{\ell',t,j} W_{\ell,s,m} \right] \right].$$

By Assumption A.5, both terms on the right hand side can be shown to be $o_P(1)$. Similarly, using Assumption A.5, again, we can show that the last term in (B.7) is $o_P(1)$. As the arguments are straightforward and tedious, we omit the details. We conclude that

$$\frac{1}{n_B} \sum_{\ell \in N_B} \left(\tilde{W}^*_{\ell,s-1} W'_{\ell,s} - \mathbf{E}_{s-2} \left[\tilde{W}^*_{\ell,s-1} W'_{\ell,s} \right] \right) = o_P(1).$$

Combined with (B.6), this delivers the desired result. ■

Lemma B.3. *As $n_B \to \infty$, $\hat{M}_w - M_{n,w} \to_P 0$.*

Proof: First we write

$$\sum_{t=2}^{T}\frac{1}{n_B}\sum_{\ell\in N_B}\left(W_{\ell,t-1}-\overline{W}_{t-1}\right)W_{\ell,t}^{\mathsf{H}\prime}=\sum_{s=2}^{T}\frac{1}{n_B}\sum_{\ell\in N_B}\tilde{W}_{\ell,s-1}W_{\ell,s}'.$$

Then, for each $s=2,\ldots,T$,

$$\frac{1}{n_B}\sum_{\ell\in N_B}\tilde{W}_{\ell,s-1}W_{\ell,s}'=\frac{1}{n_B}\sum_{\ell\in N_B}\mathbf{E}_{s-2}\left[\tilde{W}_{\ell,s-1}^{*}W_{\ell,s}'\right]+o_P(1),$$

by Lemma B.2. This gives the desired result. ■

Define

(B.8)
$$\tilde{\Omega}_{n,s}=\frac{1}{n_B}\sum_{\ell\in N_B}\sigma_{n,s}^2\tilde{W}_{\ell,s-1}^{*}\tilde{W}_{\ell,s-1}^{*\prime},\ \text{for each } s=2,\ldots,T,\ \text{and}$$
$$\Omega_n=\sum_{s=2}^{T}\Omega_{n,s}$$

and $\Omega_{n,s}$ is defined in (B.3).

Lemma B.4. *For each $s=2,\ldots,T$,*

$$\tilde{\Omega}_{n,s}-\Omega_{n,s}\to_P 0,$$

as $n_B\to\infty$.

Proof: It suffices to show that as $n_B\to\infty$,

$$\frac{1}{n_B}\sum_{\ell\in N_B}\left(\tilde{W}_{\ell,s-1}^{*}\tilde{W}_{\ell,s-1}^{*\prime}-\mathbf{E}_{s-2}\left[\tilde{W}_{\ell,s-1}^{*}\tilde{W}_{\ell,s-1}^{*\prime}\right]\right)\to_P 0.$$

The result follows from Assumption A.5, similarly as in the proof of Lemma B.2. Details are omitted for brevity. ■

Lemma B.5. *As $n_B\to\infty$,*

$$\tilde{\gamma}_{zb}-\gamma_{zb}\to_P 0.$$

Proof: We first show that $\hat{U}_{zb} = o_P(1)$. By Lemma B.1,

$$\sqrt{n_B}\hat{U}_{zb} = \sum_{s=2}^{T} \xi_s + o_P(1).$$

Note that

$$\begin{aligned}\sum_{s=2}^{T} \mathrm{Var}(\xi_s \mid \mathcal{F}_{s-1}) &= \sum_{s=2}^{T} \frac{1}{n_B} \sum_{\ell \in N_B} \sigma_{n,s}^2 \tilde{W}_{\ell,s-1}^{*} \tilde{W}_{\ell,s-1}^{*\prime} \\ &= \sum_{s=2}^{T} \frac{1}{n_B} \sum_{\ell \in N_B} \mathbf{E}_{s-2}\left[\sigma_{n,s}^2 \tilde{W}_{\ell,s-1}^{*} \tilde{W}_{\ell,s-1}^{*\prime}\right] + o_P(1) = \sum_{s=2}^{T} \Omega_s + o_P(1),\end{aligned}$$

by following the same arguments as in the proof of Lemma B.2 and Assumptions A.4 and A.6. By Assumption A.7, we have

$$\mathbf{E}\left[\sum_{s=2}^{T} \mathrm{Var}(\xi_s \mid \mathcal{F}_{s-1})\right] = O(1),$$

as $n_B \to \infty$. Since $\mathbf{E}_{s-1}[\xi_s] = 0$, for each $s = 2, ..., T$, $\xi_s = O_P(1)$, as $n_B \to \infty$. We conclude that

$$\hat{U}_{zb} = O_P(n_B^{-1/2}) = o_P(1).$$

Hence

$$\begin{aligned}\tilde{\gamma}_{zb} - \gamma_{zb} &= \left(\hat{M}_w' \hat{M}_w\right)^{-1} \hat{M}_w' \hat{U}_{zb} \\ &= \left(M_{n,w}' M_{n,w}\right)^{-1} M_{n,w}' \hat{U}_{zb} + o_P(1) = o_P(1),\end{aligned} \tag{B.9}$$

by Lemma B.3 and Assumption A.6(ii). ■

Lemma B.6. $\hat{\Omega}_{zb} - \Omega_n \to_P 0$, *as* $n_B \to \infty$.

Proof: Let $\Delta_{zb} = \tilde{\gamma}_{zb} - \gamma_{zb}$. Since $\hat{v}^{\mathsf{H}}_{\ell,t} - v^{\mathsf{H}}_{\ell,t} = -W^{\mathsf{H}\prime}_{\ell,t-1}\Delta_{zb}$, we write

$$\begin{aligned}\hat{\Omega}_{zb} &= \sum_{t=2}^{T}\sum_{t'=2}^{T}\frac{1}{n_B}\sum_{\ell\in N_B}\left(W_{\ell,t-1}-\overline{W}_{t-1}\right)\left(W_{\ell,t'-1}-\overline{W}_{t'-1}\right)'\hat{v}^{\mathsf{H}}_{\ell,t}\hat{v}^{\mathsf{H}}_{\ell,t'}\\ &= \sum_{t=2}^{T}\sum_{t'=2}^{T}\frac{1}{n_B}\sum_{\ell\in N_B}\left(W_{\ell,t-1}-\overline{W}_{t-1}\right)\left(W_{\ell,t'-1}-\overline{W}_{t'-1}\right)'v^{\mathsf{H}}_{\ell,t}v^{\mathsf{H}}_{\ell,t'}+R_n,\end{aligned}$$

where

$$\begin{aligned}R_n = &\sum_{t=2}^{T}\sum_{t'=2}^{T}\sum_{j=1}^{d_W}\sum_{j'=1}^{d_W}\Delta_{zb,j}\left(\frac{1}{n_B}\sum_{\ell\in N_B}W^{\mathsf{H}}_{\ell,t-1,j}\left(W_{\ell,t-1}-\overline{W}_{t-1}\right)\left(W_{\ell,t'-1}-\overline{W}_{t'-1}\right)'W^{\mathsf{H}}_{\ell,t'-1,j'}\right)\Delta_{zb,j'}\\ &-2\sum_{t=2}^{T}\sum_{t'=2}^{T}\sum_{j'=1}^{d_W}v^{\mathsf{H}}_{\ell,t}\left(\frac{1}{n_B}\sum_{\ell\in N_B}\left(W_{\ell,t-1}-\overline{W}_{t-1}\right)\left(W_{\ell,t'-1}-\overline{W}_{t'-1}\right)'W^{\mathsf{H}}_{\ell,t'-1,j'}\right)\Delta_{zb,j'},\end{aligned}$$

with $W^{\mathsf{H}}_{\ell,t-1,j}$ denoting the j-th entry of $W^{\mathsf{H}}_{\ell,t-1}$ and similarly with $\Delta_{zb,j}$. Note that

$$\begin{aligned}&\sum_{t=2}^{T}\sum_{t'=2}^{T}\frac{1}{n_B}\sum_{\ell\in N_B}W^{\mathsf{H}}_{\ell,t-1,j}\left(W_{\ell,t-1}-\overline{W}_{t-1}\right)\left(W_{\ell,t'-1}-\overline{W}_{t'-1}\right)'W^{\mathsf{H}}_{\ell,t'-1,j'}\\ &=\sum_{s=2}^{T}\sum_{s'=2}^{T}\frac{1}{n_B}\sum_{\ell\in N_B}W_{\ell,s-1,j}\tilde{W}_{\ell,s-1}\tilde{W}'_{\ell,s'-1}W_{i,s'-1,j'}\\ &=\frac{1}{n_B}\sum_{\ell\in N_B}\left(\sum_{s=2}^{T}W_{\ell,s-1,j}\tilde{W}_{\ell,s-1}\right)\left(\sum_{s=2}^{T}W_{i,s-1,j'}\tilde{W}_{\ell,s-1}\right)'.\end{aligned}$$

By Cauchy-Schwarz inequality, we bound the (m,m')-th entry of the last term by

$$\sqrt{\frac{1}{n_B}\sum_{\ell\in N_B}\left(\sum_{s=2}^{T}W_{\ell,s-1,j}\tilde{W}_{\ell,s-1,m}\right)^2}\times\sqrt{\frac{1}{n_B}\sum_{\ell\in N_B}\left(\sum_{s=2}^{T}W_{\ell,s-1,j}\tilde{W}_{\ell,s-1,m'}\right)^2},$$

where $\tilde{W}_{\ell,s-1,m}$ denotes the m-th entry of $\tilde{W}_{\ell,s-1}$. We write

$$\frac{1}{n_B}\sum_{\ell\in N_B}\left(\sum_{s=2}^{T}W_{\ell,s-1,j}\tilde{W}_{\ell,s-1,m}\right)^2\leq\frac{T-1}{n_B}\sum_{\ell\in N_B}\sum_{s=2}^{T}W^2_{\ell,s-1,j}\tilde{W}^2_{\ell,s-1,m}.$$

Using Assumption A.5 and following the same arguments as in the proof of Lemma B.2,

$$\begin{aligned}\frac{1}{n_B}\sum_{\ell\in N_B}\sum_{s=2}^{T}\tilde{W}_{\ell,s-1,m}^{2}W_{\ell,s-1,j}^{2} &= \frac{1}{n_B}\sum_{\ell\in N_B}\sum_{s=2}^{T}\mathbf{E}_{s-2}\left[\tilde{W}_{\ell,s-1,m}^{*2}W_{\ell,s-1,j}^{2}\right]+o_P(1)\\ &\le \sum_{s=2}^{T}\sqrt{\frac{1}{n_B}\sum_{\ell\in N_B}\mathbf{E}_{s-2}\left[\|W_{\ell,s-1}\|^4\right]}\times\sqrt{\frac{1}{n_B}\sum_{\ell\in N_B}\mathbf{E}_{s-2}\left[\|\tilde{W}_{\ell,s-1}^{*}\|^4\right]}+o_P(1).\end{aligned}$$

By Assumption A.7, we find that

$$\frac{1}{n_B}\sum_{\ell\in N_B}W_{\ell,t-1,j}^{\mathsf{H}}\left(W_{\ell,t-1}-\overline{W}_{t-1}\right)\left(W_{\ell,t'-1}-\overline{W}_{t'-1}\right)'W_{\ell,t'-1,j'}^{\mathsf{H}}=O_P(1).$$

Since $\tilde{\gamma}_{zb}=\gamma_{zb}+o_P(1)$ by Lemma B.5, the leading term in the definition of R_n is $o_P(1)$. We can deal with the second term similarly to show that it is $o_P(1)$. Hence, we have

$$\begin{aligned}&\frac{1}{n_B}\sum_{\ell\in N_B}\left(W_{\ell,t-1}-\overline{W}_{t-1}\right)\left(W_{\ell,s-1}-\overline{W}_{s-1}\right)'\hat{v}_{\ell,t}^{\mathsf{H}}\hat{v}_{\ell,s}^{\mathsf{H}}\\ &=\frac{1}{n_B}\sum_{\ell\in N_B}\left(W_{\ell,t-1}-\overline{W}_{t-1}\right)\left(W_{\ell,s-1}-\overline{W}_{s-1}\right)'v_{\ell,t}^{\mathsf{H}}v_{\ell,s}^{\mathsf{H}}+o_P(1).\end{aligned}$$

This gives us

$$\begin{aligned}\hat{\Omega}_{zb}&=\frac{1}{n_B}\sum_{\ell\in N_B}\left(\sum_{t=2}^{T}\left(W_{\ell,t-1}-\overline{W}_{t-1}\right)v_{\ell,t}^{\mathsf{H}}\right)\left(\sum_{t=2}^{T}\left(W_{\ell,t-1}-\overline{W}_{t-1}\right)v_{\ell,t}^{\mathsf{H}}\right)'+o_P(1)\\ &=\frac{1}{n_B}\sum_{\ell\in N_B}\left(\sum_{s=2}^{T}\tilde{W}_{\ell,s-1}\eta_{\ell,s}\right)\left(\sum_{s=2}^{T}\tilde{W}_{\ell,s-1}\eta_{\ell,s}\right)'+o_P(1)\\ &=\frac{1}{n_B}\sum_{\ell\in N_B}\left(\sum_{s=2}^{T}\tilde{W}_{\ell,s-1}^{*}\eta_{\ell,s}\right)\left(\sum_{s=2}^{T}\tilde{W}_{\ell,s-1}^{*}\eta_{\ell,s}\right)'+o_P(1),\end{aligned}$$

where the first equality is due to our derivation above, and the third equality follows by the same arguments in the proof of Lemma B.2.

For $s\neq s'$, we have for all $j,j'=1,\ldots,d_W$,

$$\mathbf{E}\left[\left(\tilde{W}_{\ell,s-1,j}^{*}\tilde{W}_{\ell,s'-1,j'}^{*}\right)\eta_{\ell,s}\eta_{\ell,s'}\mid\mathcal{F}_1\right]=0,$$

by Assumption A.4(i)(ii). Hence for $s \neq s'$,

$$\begin{aligned}
&\operatorname{Var}\left(\frac{1}{n_B}\sum_{\ell \in N_B} \tilde{W}^*_{\ell,s-1,j}\tilde{W}^*_{\ell,s'-1,j'}\eta_{\ell,s}\eta_{\ell,s'} \mid \mathcal{F}_1\right) \\
&= \frac{1}{n_B^2}\sum_{\ell \in N_B} \mathbf{E}\left[\left(\tilde{W}^*_{\ell,s-1,j}\tilde{W}^*_{\ell,s'-1,j'}\right)^2 \eta^2_{\ell,s}\eta^2_{\ell,s'} \mid \mathcal{F}_1\right] \\
&\leq \frac{1}{n_B}\sqrt{\frac{1}{n_B}\sum_{\ell \in N_B}\mathbf{E}\left[\left(\tilde{W}^*_{\ell,s-1,j}\eta_{\ell,s}\right)^4 \mid \mathcal{F}_1\right]}\sqrt{\frac{1}{n_B}\sum_{\ell \in N_B}\mathbf{E}\left[\left(\tilde{W}^*_{\ell,s'-1,j'}\eta_{\ell,s'}\right)^4 \mid \mathcal{F}_1\right]}.
\end{aligned}$$

Since

$$\frac{1}{n_B}\sum_{\ell \in N_B}\mathbf{E}_{s-1}\left[\tilde{W}^{*4}_{\ell,s-1,j}\eta^4_{\ell,s}\right] \leq \frac{1}{n_B}\sum_{\ell \in N_B}\tilde{W}^{*4}_{\ell,s-1,j}\max_{\ell \in N_B}\mathbf{E}_{s-1}\left[\eta^4_{\ell,s}\right] = O_P(1),$$

we find that whenever $s \neq s'$, for all $j, j' = 1, ..., d_W$,

$$\operatorname{Var}\left(\frac{1}{n_B}\sum_{\ell \in N_B} \tilde{W}^*_{\ell,s-1,j}\tilde{W}^*_{\ell,s'-1,j'}\eta_{\ell,s}\eta_{\ell,s'} \mid \mathcal{F}_1\right) = O_P\left(n_B^{-1}\right).$$

Hence

$$\begin{aligned}
\hat{\Omega}_{zb} &= \frac{1}{n_B}\sum_{\ell \in N_B}\sum_{s=2}^{T}\tilde{W}_{\ell,s-1}\tilde{W}'_{\ell,s-1}\eta^2_{\ell,s} + o_P(1) \\
&= \frac{1}{n_B}\sum_{\ell \in N_B}\sum_{s=2}^{T}\tilde{W}^*_{\ell,s-1}\tilde{W}^{*\prime}_{\ell,s-1}\sigma^2_{n,s} + o_P(1) \\
&= \frac{1}{n_B}\sum_{\ell \in N_B}\sum_{s=2}^{T}\mathbf{E}_{s-2}\left[\sigma^2_{n,s}\tilde{W}^*_{\ell,s-1}\tilde{W}^{*\prime}_{\ell,s-1}\right] + o_P(1) = \Omega_n + o_P(1),
\end{aligned}$$

by Lemma B.4. ■

For each $b \in \mathbf{R}^{d_W}$ with $b'b = 1$, we define

$$q_{n,s}(b) = \frac{\frac{1}{n_B}\sum_{\ell \in N_B}|b'\tilde{W}^*_{\ell,s-1}|^3\mathbf{E}_{s-1}\left[|\eta_{\ell,s}|^3\right]}{\left(b'\left(\frac{1}{n_B}\sum_{\ell \in N_B}\tilde{W}^*_{\ell,s-1}\tilde{W}^{*\prime}_{\ell,s-1}\sigma^2_{n,s}\right)b\right)^{3/2}}.$$

Lemma B.7. *For each $s = 1, ..., T$, and each $b \in \mathbf{R}^{d_W}$ with $b'b = 1$,*

$$q_{n,s}(b) = O_P(1),$$

as $n_B \to \infty$.

Proof: Note that

$$\frac{1}{n_B} \sum_{\ell \in N_B} |b'\tilde{W}^*_{\ell,s-1}|^3 \mathbf{E}_{s-1}\left[|\eta_{\ell,s}|^3\right] \le \max_{\ell \in N_B} \mathbf{E}_{s-1}\left[|\eta_{\ell,s}|^3\right] \frac{1}{n_B} \sum_{\ell \in N_B} |b'\tilde{W}^*_{\ell,s-1}|^3.$$

By Assumptions A.4(iii) and A.7, we have

$$\max_{\ell \in N_B} \mathbf{E}_{s-1}\left[|\eta_{\ell,s}|^3\right] \frac{1}{n_B} \sum_{\ell \in N_B} \mathbf{E}_{s-2}\left[|b'\tilde{W}^*_{\ell,s-1}|^3\right] = O_P(1),$$

as $n_B \to \infty$.

By Assumption A.6(i) and Lemma B.4, there exists $c > 0$ such that with probability approaching one as $n_B \to \infty$,

$$\lambda_{\min}\left(\frac{1}{n_B} \sum_{\ell \in N_B} \tilde{W}^*_{\ell,s-1} \tilde{W}^{*\prime}_{\ell,s-1}\right) \ge c.$$

Thus we obtain the desired result. ■

Lemma B.8. *For any vector $b \in \mathbf{R}^{d_W}$ such that $b'b = 1$, and for each $s = 2, ..., T$,*

$$\sup_{\tilde{c} \in \mathbf{R}} \left| P\left\{b'\xi_s \le \tilde{c} \mid \mathcal{F}_{s-1}\right\} - P\left\{b'\tilde{\xi}^\infty_s \le \tilde{c} \mid \mathcal{F}_{s-1}\right\}\right| = o_P(1),$$

as $n_B \to \infty$, where $\tilde{\xi}^\infty_s = \Omega_s^{1/2}\mathbb{Z}_s$, $\mathbb{Z}_s \in \mathbf{R}^{d_W}$, are i.i.d. standard normal random vectors independent of all other random variables and Ω_s is defined in Assumption A.6.

Proof: Define $\tilde{\xi}_s = \tilde{\Omega}_{n,s}^{1/2}\mathbb{Z}_s$, where $\tilde{\Omega}_{n,s}$ is as defined in (B.8). Since $\eta_{\ell,s}$, $\ell = 1, ..., n_B$, are conditionally independent given $\mathcal{F}_{s-1}$ by Assumption A.4(i) and $\tilde{\Omega}_{n,s}$ is $\mathcal{F}_{s-1}$-measurable, we use the Berry-Esseen bound (Theorem 3 of Chow and Teicher (1988), p.304) to deduce that[22]

$$\sup_{\tilde{c}\in\mathbf{R}}\left|P\left\{b'\xi_s \le \tilde{c} \mid \mathcal{F}_{s-1}\right\} - P\left\{b'\tilde{\xi}_s \le \tilde{c} \mid \mathcal{F}_{s-1}\right\}\right| \le \frac{7.5q_{n,s}(b)}{\sqrt{n_B}}.$$

The last bound is $o_P(1)$ by Lemma B.7. Furthermore, by Assumption A.6 and Lemma B.4, for each $s = 2, ..., T$,

$$\tilde{\Omega}_{n,s} = \Omega_s + o_P(1),$$

as $n_B \to \infty$. Hence

$$\begin{aligned}
&\sup_{\tilde{c}\in\mathbf{R}}\left|P\left\{b'\tilde{\xi}_s^{\infty} \le \tilde{c} \mid \mathcal{F}_{s-1}\right\} - P\left\{b'\tilde{\xi}_s \le \tilde{c} \mid \mathcal{F}_{s-1}\right\}\right| \\
&= \sup_{\tilde{c}\in\mathbf{R}}\left|P\left\{b'\tilde{\xi}_s^{\infty} \le \tilde{c} \mid \mathcal{F}_{s-1}\right\} - P\left\{b'\tilde{\xi}_s^{\infty} \le \tilde{c} + b'(\Omega_s^{1/2} - \tilde{\Omega}_{n,s}^{1/2})\mathbb{Z}_s \mid \mathcal{F}_{s-1}\right\}\right| \\
&= \sup_{\tilde{c}\in\mathbf{R}}\left|P\left\{b'\tilde{\xi}_s^{\infty} \le \tilde{c} \mid \mathcal{F}_{s-1}\right\} - P\left\{b'\tilde{\xi}_s^{\infty} \le \tilde{c} + o_P(1) \mid \mathcal{F}_{s-1}\right\}\right| = o_P(1),
\end{aligned}$$

as $n_B \to \infty$, because $b'\tilde{\xi}_s^{\infty}$ is a random variable whose conditional distribution given $\mathcal{F}_{s-1}$ is equal to that given $\mathcal{F}_1$, and its conditional distribution given $\mathcal{F}_1$ is absolutely continuous with respect to the Lebesgue measure due to Ω_s being positive definite by Assumption A.6. Hence, we obtain the desired result. ■

We set $\mathcal{G}_1 = \mathcal{F}_1$ and for each $t = 2, ..., T$, we define

$$\mathcal{G}_t = \sigma(\xi_2, \xi_3, ..., \xi_t) \vee \mathcal{F}_1,$$

where $\sigma(\xi_2, \xi_3, ..., \xi_t)$ denotes the σ-field generated by $\xi_2, \xi_3, ..., \xi_t$.

Lemma B.9. *For any vector $b \in \mathbf{R}^{d_W}$ such that $b'b = 1$ and for $\tilde{c} \in \mathbf{R}$,*

$$P\left\{\sum_{s=2}^{T} b'\xi_s \le \tilde{c} \mid \mathcal{G}_1\right\} - P\left\{\sum_{s=2}^{T} b'\tilde{\xi}_s^{\infty} \le \tilde{c} \mid \mathcal{G}_1\right\} = \sum_{s=2}^{T}\mathbf{E}\left[\Delta_{s-1}\left(\tilde{c} - b'R_{s-1}\right) \mid \mathcal{G}_1\right],$$

[22] The theorem itself is concerned with the sum of independent random variables. However, with appropriate modifications, the same bound with replacing the moments by the conditional moments given the common shock applies to a sum of conditionally independent random variables given the common shocks.

where

$$\Delta_{s-1}\left(\tilde{c}-b'R_{s-1}\right)=P\left\{b'\xi_s\leq\tilde{c}-\sum_{t=s+1}^{T}b'\tilde{\xi}_t^{\infty}-b'R_{s-1}\mid\mathcal{G}_{s-1}\right\}$$
$$-P\left\{b'\tilde{\xi}_s^{\infty}\leq\tilde{c}-\sum_{t=s+1}^{T}b'\tilde{\xi}_t^{\infty}-b'R_{s-1}\mid\mathcal{G}_{s-1}\right\}$$

and $R_s=\sum_{t=2}^{s}\xi_t$, $R_1=0$, *and* $\tilde{\xi}_s^{\infty}$, *with* $s=2,\ldots,T$, *are defined in Lemma B.8.*

Proof: First, we write

$$P\left\{\sum_{s=2}^{T}b'\xi_s\leq\tilde{c}\mid\mathcal{G}_{T-1}\right\}=P\left\{b'\xi_T\leq\tilde{c}-b'R_{T-1}\mid\mathcal{G}_{T-1}\right\}$$
$$=\Delta_{T-1}(\tilde{c}-b'R_{T-1})+P\left\{b'\tilde{\xi}_T^{\infty}\leq\tilde{c}-b'R_{T-1}\mid\mathcal{G}_{T-1}\right\},$$

where

$$\Delta_{T-1}(\tilde{c}-b'R_{T-1})=P\left\{b'\xi_T\leq\tilde{c}-b'R_{T-1}\mid\mathcal{G}_{T-1}\right\}$$
$$-P\left\{b'\tilde{\xi}_T^{\infty}\leq\tilde{c}-b'R_{T-1}\mid\mathcal{G}_{T-1}\right\}.$$

(Note that R_{T-1} is $\mathcal{G}_{T-1}$-measurable.) Hence,

$$P\left\{\sum_{s=2}^{T}b'\xi_s\leq\tilde{c}\mid\mathcal{G}_1\right\}=\mathbf{E}_1\left[\Delta_{T-1}(\tilde{c}-b'R_{T-1})\right]+P\left\{b'\tilde{\xi}_T^{\infty}\leq\tilde{c}-b'R_{T-1}\mid\mathcal{G}_1\right\}.$$

As for the last term, we write

$$P\left\{b'\tilde{\xi}_T^{\infty}\leq\tilde{c}-b'R_{T-1}\mid\mathcal{G}_{T-2}\right\}=P\left\{b'\xi_{T-1}\leq\tilde{c}-b'\tilde{\xi}_T^{\infty}-b'R_{T-2}\mid\mathcal{G}_{T-2}\right\}$$
$$=P\left\{b'\tilde{\xi}_{T-1}^{\infty}+b'\tilde{\xi}_T^{\infty}\leq\tilde{c}-b'R_{T-2}\mid\mathcal{G}_{T-2}\right\}$$
$$+\Delta_{T-2}(\tilde{c}-b'R_{T-2}).$$

Hence,

$$P\left\{\sum_{s=2}^{T} b'\xi_s \le \tilde{c} \mid \mathcal{G}_1\right\} = \mathbf{E}_1\left[\Delta_{T-1}(\tilde{c} - b'R_{T-1})\right] + \mathbf{E}_1\left[\Delta_{T-2}(\tilde{c} - b'R_{T-2})\right]$$
$$+ P\left\{b'\tilde{\xi}_{T-1}^{\infty} + b'\tilde{\xi}_{T}^{\infty} \le \tilde{c} - b'R_{T-2} \mid \mathcal{G}_1\right\}.$$

We continue this procedure until we have

$$P\left\{b'\tilde{\xi}_3^{\infty} \le \tilde{c} - b'\tilde{\xi}_T^{\infty} - b'\tilde{\xi}_{T-1}^{\infty} \cdots - b'\tilde{\xi}_4^{\infty} - b'R_2 \mid \mathcal{G}_1\right\}$$
$$= P\left\{b'\xi_2 \le \tilde{c} - b'\tilde{\xi}_T^{\infty} - b'\tilde{\xi}_{T-1}^{\infty} \cdots - b'\tilde{\xi}_3^{\infty} \mid \mathcal{G}_1\right\}$$
$$= P\left\{b'\tilde{\xi}_T^{\infty} + b'\tilde{\xi}_{T-1}^{\infty} \cdots + b'\tilde{\xi}_3 + b'\tilde{\xi}_2^{\infty} \le \tilde{c} \mid \mathcal{G}_1\right\} + \Delta_1(\tilde{c} - b'R_1),$$

where $R_1 = 0$, and

$$\Delta_1(\tilde{c} - b'R_1) = P\left\{b'\xi_2 \le \tilde{c} - b'\tilde{\xi}_T^{\infty} - b'\tilde{\xi}_{T-1}^{\infty} \cdots - b'\tilde{\xi}_3^{\infty} \mid \mathcal{G}_1\right\}$$
$$- P\left\{b'\tilde{\xi}_2^{\infty} \le \tilde{c} - b'\tilde{\xi}_T^{\infty} - b'\tilde{\xi}_{T-1}^{\infty} \cdots - b'\tilde{\xi}_3^{\infty} \mid \mathcal{G}_1\right\}.$$

By taking conditional expectations given $\mathcal{G}_1$ of all the conditional probabilities above, we obtain the desired result. ■

We let

$$\Omega_{zb} = \sum_{s=2}^{T} \Omega_s. \tag{B.10}$$

Lemma B.10. *For any vector $b \in \mathbf{R}^{d_W}$ such that $b'b = 1$, as $n_B \to \infty$,*

$$\sup_{\tilde{c} \in \mathbf{R}} \left| P\left\{b' \sum_{s=2}^{T} \xi_s \le \tilde{c} \mid \mathcal{F}_1\right\} - P\left\{b'\Omega_{zb}^{1/2}\mathbb{Z} \le \tilde{c} \mid \mathcal{F}_1\right\}\right| \to_P 0,$$

where $\mathbb{Z} \in \mathbf{R}^{d_W}$ is a standard normal random vector independent of other random variables.

Proof: By Lemma B.9,

$$\sup_{\tilde{c}\in\mathbf{R}}\left|P\left\{\sum_{s=2}^{T} b'\xi_s \le \tilde{c} \mid \mathcal{F}_1\right\} - P\left\{\sum_{s=2}^{T} b'\tilde{\xi}_s^{\infty} \le \tilde{c} \mid \mathcal{F}_1\right\}\right| \le \sum_{s=2}^{T}\mathbf{E}\left[\sup_{\tilde{c}\in\mathbf{R}}\Delta_{s-1}(\tilde{c}) \mid \mathcal{F}_1\right],$$

because R_{s-1} is $\mathcal{G}_{s-1}$-measurable. Note that for each $s = 2, ..., T$,

$$\sup_{\tilde{c}\in\mathbf{R}}\Delta_{s-1}(\tilde{c}) \le \sup_{\tilde{c}\in\mathbf{R}}\left|P\left\{b'\xi_s \le \tilde{c} \mid \mathcal{G}_{s-1}\right\} - P\left\{b'\tilde{\xi}_s^{\infty} \le \tilde{c} \mid \mathcal{G}_{s-1}\right\}\right|,$$

because $\mathbb{Z}_t$'s that constitute $\tilde{\xi}_t^{\infty}$'s are independent of $\mathcal{G}_s$, and Ω_t's are all $\mathcal{F}_1$-measurable by Assumption A.6. The last supremum is $o_P(1)$ by Lemma B.8. Since $\sup_{\tilde{c}\in\mathbf{R}}\Delta_{s-1}(\tilde{c})$ is bounded by 1, it is uniformly integrable. Hence, we find that

$$\mathbf{E}\left[\sup_{\tilde{c}\in\mathbf{R}}\Delta_{s-1}(\tilde{c}) \mid \mathcal{F}_1\right] = o_P(1),$$

for each $s = 2, ..., T$. Thus, we obtain the desired result. ■

Proof of Theorem A.1: We write

$$\sqrt{n_B}\hat{\Sigma}_{zb}^{-1/2}(\hat{\gamma}_{zb} - \gamma_{zb}) = \left(\hat{M}_w'\hat{\Omega}_{zb}^{-1}\hat{M}_w\right)^{-1/2}\hat{M}_w'\hat{\Omega}_{zb}^{-1}\sqrt{n_B}\hat{U}_{zb}. \tag{B.11}$$

By Lemma B.10, (B.11), and Cramér-Wold device, we find that

$$\Omega_{zb}^{-1/2}\sqrt{n_B}\hat{U}_{zb} \to_d N(0, I),$$

as $n_B \to \infty$, with the matrix Ω_{zb} defined in (B.10). This implies that $\sqrt{n_B}\hat{U}_{zb} = O_P(1)$.

Since $\sqrt{n_B}\hat{U}_{zb} = O_P(1)$, we use Lemmas B.3 and B.6, and Assumption A.6 to rewrite the last term in (B.11) as

$$\left(M_w'\Omega_{zb}^{-1}M_w\right)^{-1/2}M_w'\Omega_{zb}^{-1/2}\Omega_{zb}^{-1/2}\sqrt{n_B}\hat{U}_{zb} + o_P(1),$$

as $n_B \to \infty$. The leading term converges in distribution to $N(0, I)$, delivering the desired result. ■