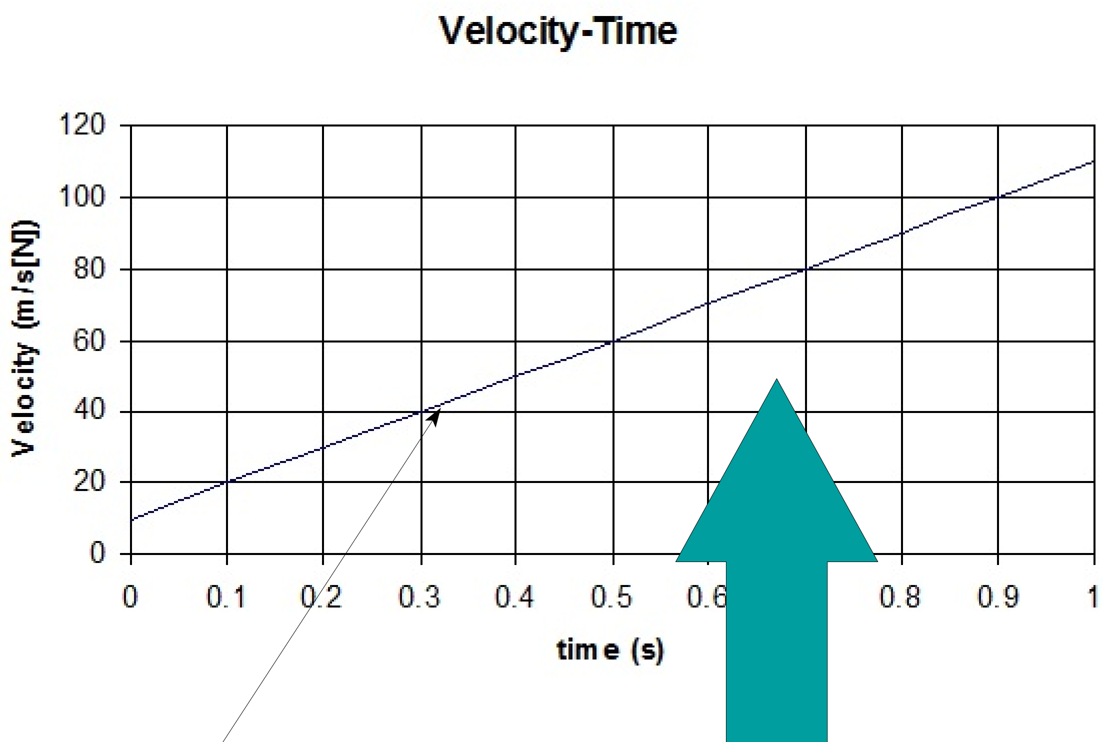
SPH3U1
Topic #6
Deriving Equations from Velocity-Time Graphs



Slope of a velocity-time graph represents acceleration.

$$a_{av} = \frac{v_2 - v_1}{\Delta t}$$

The area underneath of the line represents total displacement for specific time interval. The shape underneath the line is a trapezoid, therefore the equation we use to solve for displacement is the same as the equation for the area of a trapezoid.

$$A = \frac{(b_1 + b_2)}{2} h \quad \text{(Trapezoid)}$$

$$\Delta \vec{d} = \left(\frac{\vec{v}_1 + \vec{v}_2}{2} \right) \Delta t \quad \text{(Displacement)}$$

The equations for acceleration and displacement can be combined to form one equation. This is done through the process of substitution.

Substitution Example:

The following example substitutes the acceleration equation into the displacement equation by rearranging the acceleration equation to equal time.

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_2}{\Delta t}$$

$$\Delta t = \frac{\vec{v}_2 - \vec{v}_1}{\vec{a}}$$

$$\Delta \vec{d} = \left(\frac{\vec{v}_1 + \vec{v}_2}{2}\right)\Delta t$$

$$\Delta \vec{d} = \left(\frac{\vec{v}_1 + \vec{v}_2}{2}\right)\left(\frac{\vec{v}_2 - \vec{v}_1}{\vec{a}}\right)$$

$$\Delta \vec{d} = \frac{\vec{v}_1\vec{v}_2 - \vec{v}_1^{\,2} + \vec{v}_2^{\,2} - \vec{v}_1\vec{v}_2}{2\vec{a}}$$

$$\Delta \vec{d} = \frac{\vec{v}_2^{\,2} - \vec{v}_1^{\,2}}{2\vec{a}}$$

The above derived equation will allow you solve for displacement, acceleration, final velocity or initial velocity without the need of time to solve the problem. This is what you will encounter when solving kinematic word problems. In each question there will be a variable that can be eliminated to solve the problem. Below is a table from your textbook that outlines all the kinematic equations needed to solve problems that have **uniform acceleration**.

Table 1 Equations for Uniformly Accelerated Motion

Variables involved	General equation	Variable eliminated
$\vec{a}_{av}$, $\vec{v}_f$, $\vec{v}_i$, Δt	$\vec{a}_{av} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$	$\Delta \vec{d}$
$\Delta \vec{d}$, $\vec{v}_i$, $\vec{a}_{av}$, Δt	$\Delta \vec{d} = \vec{v}_i \Delta t + \frac{\vec{a}_{av}(\Delta t)^2}{2}$	$\vec{v}_f$
$\Delta \vec{d}$, $\vec{v}_i$, $\vec{v}_f$, Δt	$\Delta \vec{d} = \vec{v}_{av} \Delta t$ or $\Delta \vec{d} = \frac{1}{2}(\vec{v}_i + \vec{v}_f)\Delta t$	$\vec{a}_{av}$
$\vec{v}_f$, $\vec{v}_i$, $\vec{a}_{av}$, $\Delta \vec{d}$	$v_f^2 = v_i^2 + 2a_{av}\Delta d$	Δt
$\Delta \vec{d}$, $\vec{v}_f$, Δt, $\vec{a}_{av}$	$\Delta \vec{d} = \vec{v}_f \Delta t - \frac{\vec{a}_{av}(\Delta t)^2}{2}$	$\vec{v}_i$

Example Problems using Kinematic Equations.

Problem #1

Ben Rushin is waiting at a stoplight in his car. When the light turns green, Ben accelerates from rest at a rate of 6.00 m/s^2 for an interval of 4.10 seconds. Determine the displacement of Ben's car during this time period.

Given	What is the question asking for?	What variable is eliminated ?	Equation and Solution
$\vec{a} = 6.00m/s^2$ $\Delta t = 4.10s$ $\vec{v}_1 = 0$	displacement	final velocity	$\Delta d = \vec{v}_1 \Delta t + \frac{\vec{a}(\Delta t)^2}{2}$ $\Delta \vec{d} = (0)(4.10) + \frac{6.00(4.10)^2}{2}$ $\Delta \vec{d} = \frac{6.00(16.81)}{2}$ $\Delta \vec{d} = \frac{100.9}{2}$ $\Delta \vec{d} = 50.4m[Forward]$

This is the equation from the above table that does not contain final velocity.

Problem #2

Ima Hurryin approaches a stoplight in her car which is moving with a velocity of +30.0 m/s. The light turns yellow, Ima applies the brakes and skids to a stop. If Ima's acceleration is -8.00 m/s^2, determine the displacement of the car during the skidding

Given	What is the question asking for?	What variable is eliminated ?	Equation and Solution
$\vec{a} = -8.00m/s^2$ $\vec{v}_f = 0$ $\vec{v}_1 = 30.0m/s$ Note: The negative acceleration indicates the force is causing the car to slow down to a stop.	displacement	time	$\Delta \vec{d} = \frac{\vec{v}_2^2 - \vec{v}_1^2}{2\vec{a}}$ $\Delta \vec{d} = \frac{30.0^2 - 0^2}{2(8.00)}$ $\Delta \vec{d} = \frac{900 - 0}{16.0}$ $\Delta \vec{d} = 56.2m[Forward]$