1. Integration by u-substitution and Integration by partial fractions

$$\int \frac{cos(x)}{sin^3(x) + sin(x)} dx$$

Answer:

Let $u = sin(x)$, so $du = cos(x)dx$

$$= \int \frac{cos(x)}{u^3 + u} \times \frac{du}{cos(x)} = \int \frac{1}{u^3 + u} du = \int \frac{1}{u(u^2 + 1)} du = \int (\frac{A}{u} + \frac{Bu + C}{u^2 + 1}) du$$

Look at $\frac{1}{u(u^2+1)} = \frac{A}{u} + \frac{Bu+C}{u^2+1} = \frac{A(u^2+1)+(Bu+C)u}{u(u^2+1)} = \frac{Au^2+Bu^2+Cu+A}{u(u^2+1)}$

$A + B = 0$, $C = 0$, $A = 1 \Rightarrow B = -1$

So, $\int (\frac{A}{u} + \frac{Bu + C}{u^2 + 1}) du = \int (\frac{1}{u} - \frac{u}{u^2 + 1}) du = \int \frac{1}{u} du - \int \frac{u}{u^2 + 1} du$

Let $w = u^2 + 1$, so $dw = 2udu$

$$= ln|u| - \int \frac{u}{w} \times \frac{dw}{2u} = ln|u| - \frac{1}{2} \int \frac{1}{w} dw = ln|u| - \frac{ln|w|}{2} + C$$

Therefore, $\int_0^{\frac{\pi}{2}} \frac{cos(x)}{sin^3(x) + sin(x)} dx = ln|sin(x)| - \frac{ln|sin^2(x) + 1|}{2} + C$

2. Integration by parts and Integration by u-substitution

$$\int_0^{\sqrt{\pi}} xsin(x^2), \mathrm{d}x$$

Answer:

(1) Integration by parts

Let $u = x$, so $du = 1dx$

Let $dv = sin(x^2)dx$, so $v = -\frac{1}{2}cos(x^2)$

$$\int_0^{\sqrt{\pi}} xsin(x^2)dx = -x\frac{1}{2}cos(x^2) \mid_0^{\sqrt{\pi}} + \int_0^{\sqrt{\pi}} \frac{1}{2}cos(x^2)dx$$

$= -\frac{1}{2}cos(\pi) - (-x * \frac{1}{2}cos(0)) + 0 = \frac{1}{2} + \frac{1}{2} = 1$

(2) Integration by u-substitution

Let $u = x^2$, so $du = 2xdx$, so $dx = \frac{du}{2x}$

$$\int_0^{\sqrt{\pi}} xsin(x^2)dx = \int_0^{\pi} xsin(u)\frac{du}{2x} = \int_0^{\pi} \frac{sin(u)}{2} du = \frac{1}{2} \int_0^{\pi} sin(u)du$$

$= \frac{1}{2}(-cos(u)) \mid_0^{\pi} = \frac{1}{2}(-cos(\pi)) - \frac{1}{2}(-cos(0)) = \frac{1}{2} * 1 - \frac{1}{2}(-1)$

$= \frac{1}{2} + \frac{1}{2} = 1$

3. Integration by partial fraction and Integration by u-substitution

$$\int_{-4}^{4} \frac{1}{(x + 5)(x^2 + 3)} dx$$

$$\frac{1}{(x+5)(x^2+3)} = \frac{A}{(x^2+3)} + \frac{Bx+C}{(X^2+3)}$$
$$A(x^2+3) + (Bx+C)(x+5) = 1$$
$$Ax^2 + 3A + Bx^2 + 5Bx + Cx + 5C = 1$$
$$A + B = 0,\ 5B + C = 0,\ 3A + 5C = 1$$
$$B = -A$$

Solving the system of equations

$$3A + 5C = 1$$
$$25A - 5C = 0$$
$$A = \frac{1}{28},\ B = -\frac{1}{28},\ C = \frac{5}{28}$$
$$\frac{1}{28}\int_{-4}^{4} \frac{1}{(x+5)}dx - \frac{1}{28}\int_{-4}^{4} \frac{x}{(x^2+3)}dx + \frac{5}{28}\int_{-4}^{4} \frac{1}{(x^2+3)}$$

First, solve $\frac{1}{28}\int_{-4}^{4} \frac{1}{(x+5)}dx$

Let $u = x+5$, $du = 1dx$

$$\frac{1}{28}\int_{1}^{9} \frac{1}{u}du = \frac{1}{28}(ln|9|)$$

Second, solve $-\frac{1}{28}\int_{-4}^{4} \frac{x}{(x^2+3)}dx$

Let $u = x^2+3$, $du = 2xdx$

$$-\tfrac{1}{28}\int_{19}^{19} \frac{x}{(u)2x}du = -\frac{1}{28}\int_{19}^{19} \frac{1}{(u)}du = -\frac{1}{28}(ln|19| - ln|19|) = 0$$

Third, solve $\frac{5}{28}\int_{-4}^{4} \frac{1}{(x^2+3)}dx$

Let $u = \frac{x}{\sqrt{3}}, du = \frac{1}{\sqrt{3}}dx$

$$\frac{5}{28}\int_{-\frac{4}{\sqrt{3}}}^{\frac{4}{\sqrt{3}}} \frac{1}{(u^2+1)\sqrt{3}}du = \frac{5}{28\sqrt{3}}\int_{-\frac{4}{\sqrt{3}}}^{\frac{4}{\sqrt{3}}} \frac{1}{(u^2+1)}du = \frac{5}{28\sqrt{3}}(arctan(u))|_{-\frac{4}{\sqrt{3}}}^{\frac{4}{\sqrt{3}}}$$
$$= \frac{5}{28\sqrt{3}}(arctan\frac{4}{\sqrt{3}} - arctan(-\frac{4}{\sqrt{3}}))$$

Therefore, $\int_{-4}^{4} \frac{1}{(x+5)(x^2+3)}dx = \frac{ln|9|}{28} + \frac{5}{28\sqrt{3}}(arctan\frac{4}{\sqrt{3}} - arctan(-\frac{4}{\sqrt{3}}))$

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