First Name: ______________ Last Name: ______________

Student-No: ______________ Section: ______________

Very short answer questions

1. 2 marks Each part is worth 1 mark. Please write your answers in the boxes.

   (a) Compute $\lim_{x\to 2} \frac{x^2\cdot e^{3\cdot x}+x\cdot\ln(x)}{\sqrt{x+1}}$.

   Answer: $\frac{4\cdot e^6+2\cdot\ln(2)}{\sqrt{3}}$

   Solution: $\lim_{x\to 2} \frac{x^2\cdot e^{3\cdot x}+x\cdot\ln(x)}{\sqrt{x+1}} = \frac{4\cdot e^6+2\cdot\ln(2)}{\sqrt{3}}$.

   (b) What is the future value of \$250 invested for 8 months at a nominal interest rate of 4% compunded monthly?

   Answer: $\$250 \cdot \left(1+\frac{0.04}{12}\right)^8$

   Solution: Direct substitution into $FV = PV \cdot \left(1+\frac{i}{n}\right)^{nt}$, where $PV = \$250$, $n = 12$, $t = \frac{8}{12} = \frac{2}{3}$ and $i = 4\% = 0.04$.

Short answer questions — you must show your work

2. 4 marks Each part is worth 2 marks.

   (a) Compute $\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9}$.

   Answer: $\frac{1}{6}$

   Solution: We have

   $$\frac{\sqrt{x}-3}{x-9} = \frac{(\sqrt{x}-3)\cdot(\sqrt{x}+3)}{(x-9)\cdot(\sqrt{x}+3)} = \frac{x-9}{(x-9)\cdot(\sqrt{x}+3)} = \frac{1}{\sqrt{x}+3}$$

   and hence

   $$\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9} = \lim_{x\to 9} \frac{1}{\sqrt{x}+3} = \frac{1}{6}.$$

   (b) You receive a loan with real interest rate of 2%, what is the nominal interest rate assuming it was compounded quarterly.

   Answer: $i = 4\cdot(e^{0.005}-1)$

   Solution: Substitute into the formulas for compounded interest and for continuous interest:

   $$PV\cdot e^{rt} = FV = PV\cdot(1+i/n)^{nt}, \quad r = 0.02, \quad n = 4, \quad i = ?$$

and the simplifying:

$$\begin{aligned}
e^{0.02t} &= (1+i/4)^{4t} \\
0.02t &= 4t \cdot \ln\left(1+\frac{i}{4}\right) \\
\frac{0.02}{4} &= \ln\left(1+\frac{i}{4}\right) \\
e^{\frac{0.02}{4}} &= 1+\frac{i}{4} \\
i &= 4 \cdot \left(e^{0.005}-1\right).
\end{aligned}$$

Long answer question — you must show your work

3. $\boxed{\text{4 marks}}$ An umbrella factory sells 6,000 umbrellas a year at the cost of \$8 a piece. An umbrella sale last December showed that a decrease of \$1 in the price of an umbrella caused an increase in selling of 500 umbrellas a month.

   Find the linear demand equation for the umbrellas. Use the notation p for price and q for the monthly demand.

   (a) Answer: $q = -500 \cdot p + 10,000$

Solution: The linear demand curve is given by $q = A \cdot p + B$. The data we are given implies that

$$\begin{aligned}
6,000 &= A \cdot 8 + B \\
6,500 &= A \cdot 7 + B
\end{aligned}$$

Substracting the two equations yield $A = -500$. Plugging A into the first equation we get

$$6,000 = -500 * 8 + B$$

and hence $B = 10,000$.

(b) Compute the maximal revenue of the factory.

Answer: \$50,000

Solution: The revenue is given by $R = p \cdot (-500 \cdot p + 10,000)$ whose zeros are at $p = \$0$ and $p = \$20$ and hence the maximal revenue will be \$50,000 when the price is \$10 a piece. In this case they sell 5,000 umbrellas a month.