Introductory example: We, Today will solve a simple version of a standard problem in economy. But first, a few definitions.

Def: • The revenue is the amount of money (R) that a company recieves by selling q items at a set price p.

$$\boxed{R = p \cdot q}$$

$\begin{cases} p = 2\$ \\ q = 10 \end{cases}$ $R = 20\$$, $R = 2q$

• The cost is the amount of money (C) a company spends to make q items.

$$\boxed{C(q) = F + V(q)}$$

F: Fixed costs: rent, comercials, salaries, etc.

$V(q)$: Variable costs: materials etc.

$F = 100$
$V(q) = 1.5 \cdot q$
$C(q) = 100 + 1.5 \cdot q$

$$\boxed{F = C(0)}$$

• The profit is the amount of money (P) the company is left with once all products are sold and all costs are paid.

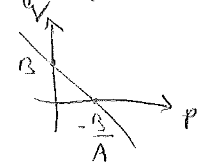
$$\boxed{P = R - C}$$

$P = 2q - 100 - 1.5q$

• Demand is the relation between the price of an item and the number of items that will be sold at that price.

A Basic principle of economy: increase in price leads to decrease in demand
$\Downarrow$
$(A < 0)$

Today we assume linear demand, namely $q = Ap + B$



What is the main goal of a good buisness?

1. Maximize revenue.
2. Minimize cost.
3. Maximize profit
4. Maximize demand.

How to price your merchandise?

We were hired by BChalk.

They are selling a chalk box for 2\$ and sell 3,000 boxes a month.

Last April they had a chalk sale and after a discount of 10¢ they sold a 100 more boxes.

- Demand: Write $q = Ap + B$.

We have:
$$\begin{cases} 3{,}000 = A \cdot 2 + B \\ A \cdot 1.9 + B = (A \cdot 2 + B) + 100 \end{cases}$$

this implies $\Rightarrow$
$$-0.1 \cdot A = 100 \Rightarrow \boxed{A = -1{,}000}$$

and $3{,}000 = -1{,}000 \cdot 2 + B$

$\Rightarrow \boxed{B = 5{,}000}$ ← (how many object we sell for free.)

- Revenue: $q = -1{,}000 \cdot p + 5{,}000 \Rightarrow q - 5{,}000 = -1{,}000 \cdot p$

$$\Rightarrow \boxed{p = \frac{q - 5{,}000}{-1{,}000}}$$

$$\boxed{R(q) = \frac{(q - 5{,}000)\, q}{-1{,}000}}$$ quad. eq. (graph it)

The fixed cost of running BChalk is 3,250 \$ a month and it costs an extra 75¢ to make a box.
So now their monthly profit 500 \$ and they wish to improve it.

- Cost: $C(q) = F + \Delta q = 3{,}250 + 0.75q$.

- Profit: $P(q) = R(q) - C(q) = \dfrac{q^2 - 5{,}000q}{-1000} - (3{,}250 + 0.75q)$

$$\boxed{P(q) = \frac{q^2 - 5{,}000q + 3{,}250{,}000 + 750q}{-1{,}000} = \frac{q^2 - 4{,}250q + 3{,}250{,}000}{-1{,}000}}$$

<u>Def</u>: The break-even points are the values of q for which $R(q) = C(q)$; alternatively $P(q) = 0$.

<u>Break-even points</u>: $2{,}125$

$$P(q) = 0 \Rightarrow q^2 - 4250q + 3{,}250{,}000 = 0$$

completing the square $\Rightarrow (q - 2{,}125)^2 - 4{,}515{,}625 + 3{,}250{,}000 = 0$

$((a \pm b)^2 = a^2 \pm 2ab + b^2)$

$$\Rightarrow (q - 2{,}125)^2 = 1{,}265{,}625 = (1{,}125)^2$$

$\boxed{a^2 \pm 2ab = (a \pm b)^2 - b^2}$

$$\Rightarrow \boxed{q_{1,2} = 2{,}125 \pm 1{,}125} = 3{,}250,\ 1{,}000$$

<u>Max. Profit</u>: This is the maximum of the parabola $P(q)$ which is in the middle between q_1 and q_2.

Namely, at $q_0 = \frac{q_1 + q_2}{2} = 2{,}125$ boxes a month.

What is the right price?

$$p_0 = \frac{q_0 - 5{,}000}{-1{,}000} = 2.875\$.$$

For more complicated models of demand we need: CALCULUS!

$P(q) = 1{,}265.625\$$ a month (more then double