# Exercises:

ⓐ You burrow \$50,000 from Nick the shark, who charges you at a fixed rate r that is compounded continuously. If you pay Nick \$100,000 two years later, what was the annual rate of interest that he charged? What was the effective interest rate? Nominal?

Solution: $FV = PV \cdot e^{rt}$ 

calculator ready form

$FV = PV(1+jt)$

$100{,}000 = 50{,}000(1+j \cdot 2)$

$2 = 1+2j$

$\boxed{j = \frac{1}{2}}$

$100{,}000 = 50{,}000 \cdot e^{2r}$ $(t=2,\ FV = 100{,}000,\ PV = 50{,}000)$

$2 = e^{2r}$

$\ln 2 = 2r$

$\boxed{r = \frac{\ln 2}{2}} \approx 0.347\ldots = 34.7\%$

other options: $\frac{\ln(\frac{1}{2})}{2}$, $\frac{1}{2}$, $\sqrt{2}-1$, don't know

$FV = PV(1+\frac{i}{n})^{n \cdot t} \rightsquigarrow 100{,}000 = 50{,}000(1+i)^2$

$2 = (1+i)^2$ ; $1+i = \sqrt{2}$ ; $\boxed{i = \sqrt{2}-1} \approx 0.414\ldots \approx 41.4\%$

ⓑ How many years will it take for \$10,000 to grow to \$12,000 if it is invested at 12% annual interest compounded quarterly?

$FV = PV(1+\frac{i}{n})^{n \cdot t}$

$12{,}000 = 10{,}000\left(1+\frac{0.12}{4}\right)^{4t}$

$1.2 = \left(1+\frac{0.12}{4}\right)^{4t}$

$\ln(1.2) = \ln\left[\left(1+\frac{0.12}{4}\right)^{4t}\right] = 4t\ln\left(1+\frac{0.12}{4}\right)$

$\boxed{t = \frac{\ln(1.2)}{4\ln(1+\frac{0.12}{4})}} \approx 18$ and a half months

other options: $\frac{\ln(1.2)}{\ln(1+\frac{0.12}{4})}$, $\underbrace{\frac{\ln(1.2)}{\ln(1.12)}}_{\frac{0.2}{0.12}}$, don't know

(c) What continuously compounded rate is equivalent to 8% compounded semi-annually (every $\frac{1}{2}$ a ~~month~~ year)

Solution:

$$\cancel{PV}\cdot e^{rt} = FV = \cancel{PV}\left(1+\frac{0.08}{2}\right)^{2t}$$

$$e^{rt} = (1+0.04)^{2t}$$

$$r\cdot\cancel{t} = \ln\{(1+0.04)^{2t}\} = 2\cancel{t}\ln(1+0.04)$$

$$\boxed{r = 2\ln(1.04)}$$

other options: $2(e^{0.04}-1)$

$e^{0.04}-1$

$\ln(1.04)$

don't know

(d) (A$^+$ problem)

A student has a trust currently valued at \$500,000 gets an annual interest rate of 6% compounded continuously. Their plan is to retire once they can withdraw \$10,000 at the beginning of each month indefinitely. How long do they need to wait to retire assuming they don't invest any more money?

Solution:

Step 1: How much money (PV) they need at the time of retirement

$\underbrace{FV}_{\text{in a month}} = PV + 10{,}000;$ $FV = PV\,e^{0.06\cdot\frac{1}{12}}$

$$PV + 10{,}000 = PV\cdot e^{0.06\cdot\frac{1}{12}}$$

$$PV\cdot\left(e^{0.06\cdot\frac{1}{12}}-1\right) = 10{,}000$$

$$\boxed{PV = \frac{10{,}000}{e^{0.005}-1}} \approx 1{,}995{,}004.11$$

Step 2:

$$500{,}000\,e^{0.06t} = \frac{10{,}000}{e^{0.005}-\cancel{1}}$$

$$e^{0.06t} = \frac{1}{50(e^{0.005}-\cancel{1})}$$

$\frac{6}{100}$

$$0.06t = \ln\left[\frac{1}{50(e^{0.005}-1)}\right]$$

$$\boxed{t = \frac{100}{6}\ln\left[\frac{1}{50(e^{0.005}-1)}\right]}$$

$\approx 23$ years.