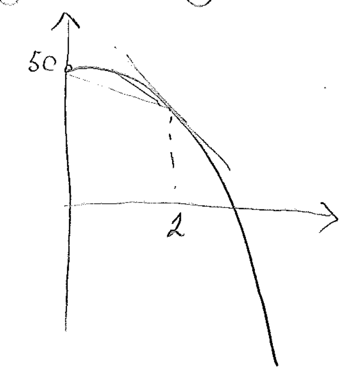
Rates of change - Introduction to derivatives

We started the discussion of limits by considering the example of free fall.

Height, $f(t) = -5t^2 + 50$.



Average speed at $[t,2]$ (or $[2,t]$) $\quad \dfrac{f(t)-f(2)}{t-2}$ $\quad$ slope of lines

Instantaneous speed $\quad \displaystyle\lim_{t\to 2} \frac{f(t)-f(2)}{t-2} = -20$ $\quad$ slope of tangent

Now, more generally: Assume $f(x)$ is defined in a neighborhood of $x=a$

Average rate of change $\quad \dfrac{f(x)-f(a)}{x-a}$

Instantaneous rate of change $=$ Derivative $\quad \boxed{\displaystyle\lim_{x\to a} \frac{f(x)-f(a)}{x-a} = f'(a)}$. slope of the tangent at $(a, f(a))$

$l(x) = f'(a)(x-a) + f(a)$

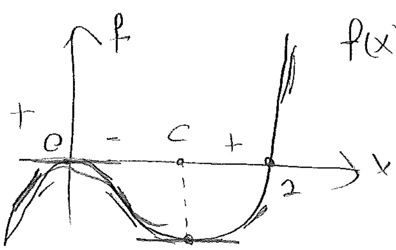
Example: Given $f(x) = 3x^2 + 5x - 7$, find $f'(1)$.

Solution: $\displaystyle f'(1) = \lim_{x\to 1} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1} \frac{(3x^2+5x-\cancel{7}) - (3\cdot 1^2 + 5\cdot 1 - \cancel{7})}{(x-1)}$

$\displaystyle = \lim_{x\to 1} \frac{3(x^2-1^2) + 5\cdot(x-1)}{(x-1)} = \lim_{x\to 1}\left[3(x+1)+5\right] = 3\cdot 2 + 5 = 11.$

HW: Compute $f'(3)$.

Ex: Draw a sketch of f' from the given $f(x)$ graph.



$f(x) = x^3 - 2x^2 = x^2(x-2)$