Warm-Up:

Compute the derivatives of the following functions:

① $f(x) = 7X^2 - 5X + 7$

Sol: $f'(x) = 14X - 5$

② $f(r) = e^{3\cdot r}$

$$g(x) = e^x \quad g'(x) = e^x$$
$$h(r) = 3r \quad h'(r) = 3$$

$$f'(r) = \frac{df}{dr}$$

$$f'(r) = (g \circ h)'(r) =$$
$$= g'(h(r)) \cdot h'(r) = e^{3r} \cdot 3$$

③ $f(x) = \dfrac{x^2+3}{X}$

$$f'(x) = \frac{(x^2+3)' \cdot x - (x)' \cdot (x^2+3)}{x^2} = \frac{2x \cdot x - 1 \cdot (x^2+3)}{x^2} = \frac{2x^2 - x^2 - 3}{x^2} =$$
$$= \frac{x^2-3}{x^2}$$

④ $f(t) = \dfrac{t^2-1}{t+1}$

$$f'(t) = \frac{(t^2-1)'(t+1) - (t^2-1)\cdot(t+1)'}{(t+1)^2} = \frac{2t(t+1) - (t^2-1)\cdot 1}{(t+1)^2} = \frac{2t^2+2t-t^2+1}{(t+1)^2} =$$
$$= \frac{t^2+2t+1}{(t+1)^2} = \frac{(t+1)^2}{(t+1)^2} = 1$$

How did that happened?

$$\frac{t^2-1}{t+1} = \frac{(t-1)\cancel{(t+1)}}{\cancel{t+1}} = t-1.$$

(5) $f(t) = \dfrac{e^{3t} \cdot t^2 + \ln(t)}{t^2 + e^{-t}}$

$$f'(t) = \frac{(e^{3t} \cdot t^2 + \ln(t)^2)'(t^2 + e^{-t}) - (e^{3t} \cdot t^2 + \ln(t)^2)(t^2 + e^{-t})'}{t^2 + e^{-t}} =$$

$$= \frac{\left(3e^{3t} \cdot t^2 + e^{3t} \cdot 2t + 2\ln(t) \cdot \frac{1}{t}\right)(t^2 + e^{-t}) - (e^{3t} \cdot t^2 + \ln(t)^2)(2t - e^{-t})}{(t^2 + e^{-t})}.$$

(6) $g(x) = e^2\left(\sin\left(\frac{\pi}{4}\right) - 1\right)$

$g'(x) = 0$

(7) $f(x) = g(x)^n$ (not that $g(x)$)

$f'(x) = n\, g(x)^{n-1} \cdot f'(x)$ By the chain rule

$f(x) = h(g(x))$ $\quad h(x) = x^n$

$h'(x) = nx^{n-1}$.

$\boxed{l(x) = f'(a)(x-a) + f(a)}$

Tangents:

(1) Find the tangent of $f(x) = e^{x^2}$ at $x = 1$.

$f'(x) = 2xe^{x^2}$

$f'(1) = 2e$ $\quad \boxed{l(x) = 2e(x-1) + e = 2ex - e}$

(2) Find a value of a such that the tangent of $f(x) = \ln(x^2+a)$ at $x=1$ pass through $(2,2)$:

Solution: $f'(x) = \frac{1}{x^2+a} \cdot 2x = \frac{2x}{x^2+a}$

$f'(1) = \frac{2}{1+a}$ $\qquad$ $f(1) = \ln(1+a)$

$l(x) = \frac{2}{1+a}(x-1) + \ln(1+a)$

$l(2) = \frac{2}{1+a}(2-1) + \ln(1+a)$

$= \frac{2}{1+a} + \ln(1+a) \overset{!}{=} 2$

$\boxed{a=0}$ solves the eq.