Closed Interval Method

Let f be a cont. func. on $[a,b]$

EVT: f has an absolute max. and min. in $[a,b]$

absolute extremum / global extremum.

Weierstrass theorem.

The idea of how to find them:

① Look for critical points at (a,b): $c_1, \dots, c_n$

(either $f'(c)=0$ or $f'(c)$ doesn't exist)

usually it's a finite number of points

② Compute values at critical points $(f(c_1), \dots f(c_n))$ and at edges $f(a)$, $f(b)$

③ Compare

Examples:

① Find extreme values of $f(x) = 2x^3 + 3x^2 - 12x$ on $[0,3]$.

sol:

$$f'(x) = 6x^2 + 6x - 12$$

$$f'(c) = 0 \longrightarrow \cancel{6}c^2 + \cancel{6}c - \overset{2}{\cancel{12}} = 0$$

$$c^2 + c - 2 = 0$$

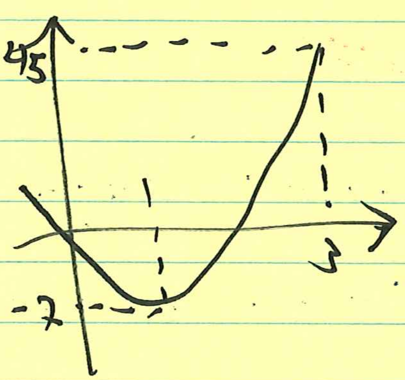
$$c = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2} \begin{cases} -2 \\ 1 \end{cases}$$

critical points: 1

Values of function: $f(0) = 0$

$f(1) = -7 \longleftarrow$ Abs. Min.

$f(3) = 45 \longleftarrow$ Abs. Max.



Find extreme values of:

② $f(x) = \begin{cases} (x-1)^2, & x \le 0 \\ x+1, & x > 0 \end{cases}$ at $[-4,2]$

$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} (x+1) = 1$ So $f(x)$ is cont. on $[-4,2]$.

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$\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} (x-1)^2 = 1$

EVT $\longrightarrow$ There are extreme values.

$f'(x) = \begin{cases} 2(x-1) & x<0 \\ 1 & x>0 \end{cases}$

$f'(0) = ?$

$$\lim_{x\to 0^+} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0^+} \frac{(x+1)-1}{x-0} = \lim_{x\to 0^+} \frac{\overset{1}{\cancel{x}}}{\cancel{x}} = 1$$

$$\lim_{x\to 0^-} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0^-} \frac{\overset{x^2-2x+1}{(x-1)^2}-1}{x-0}$$

$$= \lim_{x\to 0^-} \frac{x^2-2x}{x} = \lim_{x\to 0^-} (x-2) = -2$$

$f'(0)$ doesn't exist!

$$f'(c) = 0 \nearrow 2(c-1) = 0, \ c<0 \longrightarrow \cancel{c = 1}$$

$$\searrow \cancel{1 = 0, \ c>0}$$

critical points: $c = 0$.

values: $f(-4) = 25 \longleftarrow$ Abs. Max.

$f(0) = 1 \longleftarrow$ Abs. Min.

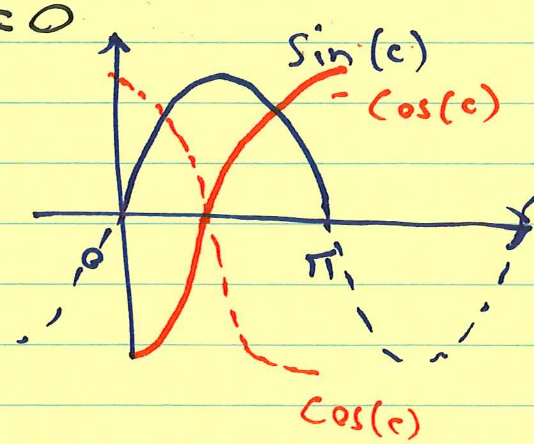
$f(2) = 3$

③ Find extreme values of $f(x) = e^x \sin x$ on $[0, \pi]$.

Sol: $f'(x) = e^x \sin x + e^x \cos x = e^x (\sin x + \cos x)$

$f'(c) = 0 \longrightarrow e^c (\sin(c) + \cos(c)) = 0$

$\longrightarrow \sin(c) + \cos(c) = 0$



	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
Sin	$0 = \frac{\sqrt{0}}{2}$	$\frac{1}{2} = \frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$1 = \frac{\sqrt{4}}{2}$
Cos	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$

$\boxed{c = \frac{3\pi}{4}}$

critical point.

$-\cos(c) = \cos(\pi - c)$

$f(0) = 0 \longleftarrow$ Abs. Min.

$f\left(\frac{3\pi}{4}\right) = e^{\frac{3\pi}{4}} \frac{\sqrt{2}}{2} \longleftarrow$ Abs. Max

$f(\pi) = 0 \longleftarrow$ Abs. Min.

$$[0, \pi]$$

$$\cos(X) = \sin(X) \longrightarrow X = \frac{\pi}{4}$$

$$-\cos(X) = \sin(X)$$

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$$\cos(\pi - X) = \sin(\pi - X)$$

$$\pi - X = \frac{\pi}{4} \longrightarrow X = \frac{3\pi}{4}$$