Math 104 section 108 Quiz #4 Column C   Date: Grade:

First Name: ____________________ Last Name: ____________________

Student-No: ____________________ Section: ____________________

Short answer questions — you must show your work

1. [4 marks] Each part is worth 2 marks.

   (a) The cost to make q TVs in our factory is given by $C(q) = 700,000 + 200\sqrt{q}$. If we make 10,000 TVs, what are the marginal unit cost, marginal cost and average cost of making one more TV?

   Solution:

   - Marginal unit cost:

   $$MUC(10,000) = C(10,001) - C(10,000) = 200\left(\sqrt{10,001} - \sqrt{10,000}\right).$$

   - Average cost $AC(10,000) = \frac{C(10,000)}{10,000} = \frac{700,000+200\sqrt{10,000}}{10,000} = 72$
   - Marginal cost

   $$MC(10,000) = C'(10,000) = \frac{100}{\sqrt{10,000}} = \frac{100}{100} = 1,$$

   since $C'(q) = 200\frac{1}{2\sqrt{q}} = \frac{100}{\sqrt{q}}$.

   (b) The distance, in meters, of a cream-pie from a clowns face is given $d(t) = -t^2 - t + 2$ (time is measured in seconds). What is the velocity in which it hits the clowns face?

   Answer: $-3\frac{m}{sec}$

   Solution: We start by solving $d(t_0) = 0$. There are two solutions, -2 and 1. Since -2 doesn't make any sense we find that $t_0 = 1$. The velocity at time 1 is $d'(2) = -2 \cdot 1 - 1 = -3\frac{m}{sec}$

Long answer questions — you must show your work

2. [6 marks] Each part is worth 3 marks.

   (a) The demand equation for a certain product is $q^2 + p^{3/2} + 3p = 70$, where q is the number of units per hour the manufacturer can sell at a price of p dollars per unit. If the price is raised slightly from \$9 dollars, will the revenue increase or decrease (use the elasticity of demand to do this)?

   Answer: Decrease

Solution: We first find $\frac{dq}{dp}$. Differentiating the demand equation with respect to p yields

$$2q\frac{dq}{dp} + \frac{3}{2}\sqrt{p} + 3 = 0$$

So

$$\frac{dq}{dp} = -\frac{3\sqrt{p}+6}{4q}$$

The elasticity is given by

$$\epsilon = \frac{p}{q}\frac{dq}{dp} = -\frac{p\left(3\sqrt{p}+6\right)}{4q^2}.$$

On the other hand, when $p = \$4$ we have $q^2 + 27 + 27 = 70$, i.e. $q = \sqrt{16} = 4$. Plugging this into the elasticity function yields

$$\epsilon = -\frac{9\left(3\sqrt{9}+6\right)}{4 \cdot 4^2} = -\frac{105}{64} < -1$$

Hence the good is price elastic and hence the revenue will decrease if the price is increased.

(b) Find the absolute maximum and minimum of $f(x) = x^{2/3} + x$ on $[-1, 1]$ (value and point).

Answer: Max 2, Min 0

Solution: We derive the function

$$f'(x) = \frac{2}{3x^{1/3}} + 1,$$

this is true for any $x \neq 0$. Solving the equation $f'(c) = 0$ for c yields one solution $c = -\left(\frac{2}{3}\right)^3$ which is in the interval. We have two critical points 0 and $-\left(\frac{2}{3}\right)^3$. We plug the edges and critical points into the function

$$f(-1) = 0, \quad f(0) = 0, \quad f\left(-\left(\frac{2}{3}\right)^3\right) = \frac{4}{27}, \quad f(1) = 2$$

The absolute maximum is 2 at 1 and the absolute minimum is 0 at -1 and 0.