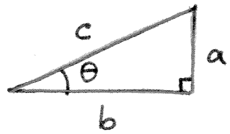
Trig Cheatsheet

Right Triangle :



$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$ $\Rightarrow$ $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{a}{b}$

$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{b}{a}$

Important identity :

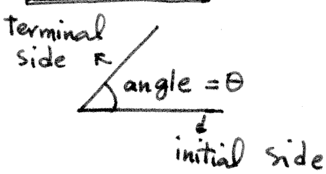
$\sin^2\theta + \cos^2\theta = 1$

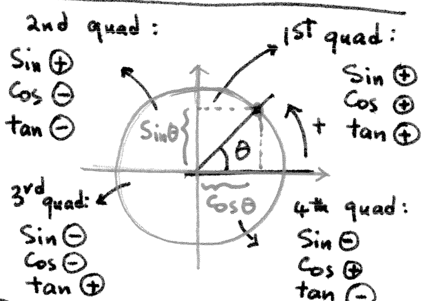
other forms

$\sin^2\theta = 1 - \cos^2\theta \Rightarrow \sin\theta = \pm\sqrt{1-\cos^2\theta}$

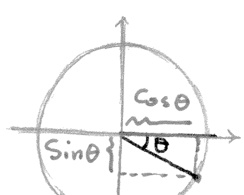
$\cos^2\theta = 1 - \sin^2\theta \Rightarrow \cos\theta = \pm\sqrt{1-\sin^2\theta}$

Unit circle





- initial side goes to positive x-axis, in counter clock-wise direction move by θ and place the terminal side
  ⇝ This is the positive direction.
- terminal side of the angle intersects the circle, the x-coordinate of this point is $\cos\theta$ and its y-coordinate is $\sin\theta$.
- If you move from positive x-axis (0) and go in clockwise direction, the angle will be Negative.



⇝ This is the negative direction.

Angles in Radian :

$\frac{\text{degree}}{180} \times \pi = \text{radian}$

deg	0	30°	45°	60°	90°	180°	270°	360°
rad	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$3\pi/2$	2π

Special Angles

Memorize Sin, Cos and tan for these angles :

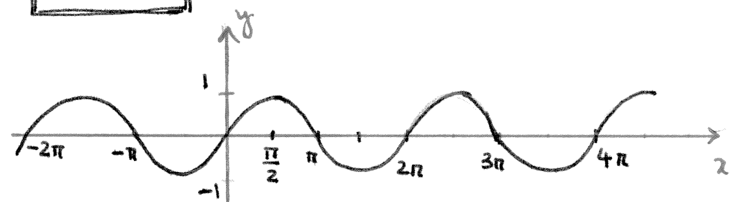
θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$3\pi/2$	2π
$\sin\theta$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1	0	-1	0
$\cos\theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0	-1	0	1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	DNE	0	DNE	0

Graphs :

$\boxed{y = \sin x}$

x-int : $0, \pm\pi, \pm 2\pi, \pm 3\pi$

⇝ $n\pi$ $n = 0, \pm 1, \pm 2, \ldots$



Domain $= (-\infty, \infty)$

Range $= [-1, 1]$

Cont's everywhere

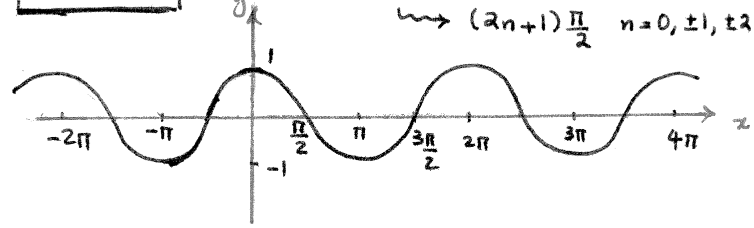
period $= 2\pi$ ⇝ $\sin(x + 2\pi) = \sin x$

⇝ Repeats itself every 2π

$\boxed{y = \cos x}$

x-int : $\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$

⇝ $(2n+1)\frac{\pi}{2}$ $n = 0, \pm 1, \pm 2, \ldots$



Domain $= (-\infty, \infty)$

Range $= [-1, 1]$

Cont's everywhere

Period $= 2\pi$ ⇝ $\cos(x + 2\pi) = \cos x$

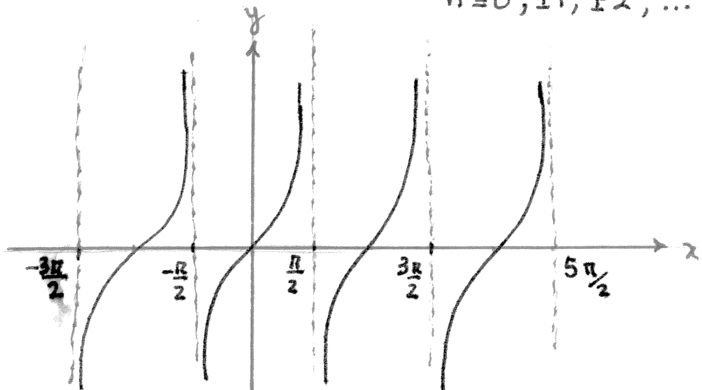
$$\boxed{y = \tan x = \frac{\sin x}{\cos x}}$$

it's NOT defined when $\cos x = 0$

i.e. when $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$

or $x = (2n+1)\frac{\pi}{2}$

when $n = 0, \pm 1, \pm 2, \ldots$



Domain $= (-\infty, \infty)$ except $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$

Range $= (-\infty, \infty)$

Points of discontinuity : $x = (2n+1)\frac{\pi}{2}$

$n = 0, \pm 1, \pm 2, \ldots$

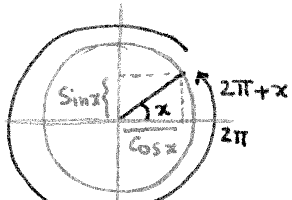
Period $= \pi \rightsquigarrow \tan(x+\pi) = \tan x$

Symmetries $\rightsquigarrow$ Always locate the angle in the circle first.

(Use unit circle)

$\sin(x + 2\pi) = \sin x$

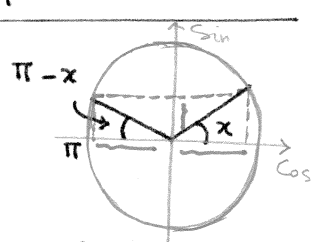
$\cos(x + 2\pi) = \cos x$



$x + 2\pi$: a complete rotation $+ x$ (1st quadrant)

$\sin(\pi - x) = \sin x$

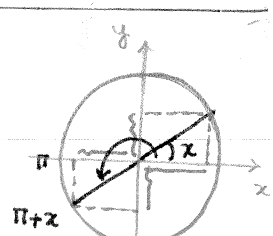
$\cos(\pi - x) = -\cos x$



$\pi - x$: above π (2nd quadrant)

$\sin(\pi + x) = -\sin x$

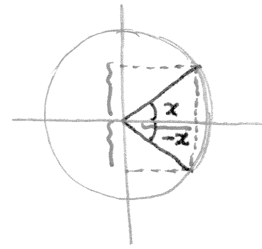
$\cos(\pi + x) = -\cos x$



$\pi + x$: below π (3rd quadrant)

$\sin(-x) = -\sin x$

$\cos(-x) = \cos x$



$-x$: Negative direction (4th quadrant)

Example. Find

$$\underbrace{\sin\left(\frac{3\pi}{4}\right)}$$

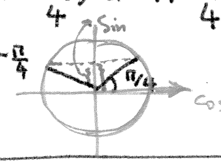
Break this into two pieces such that one part is π or 2π and the other part is one of the special angles that you must memorize :

$$\frac{3\pi}{4} = \frac{4\pi - \pi}{4} = \frac{4\pi}{4} - \frac{\pi}{4} = \pi - \frac{\pi}{4}$$

$$\Rightarrow \sin\left(\frac{3\pi}{4}\right) = \sin\left(\pi - \frac{\pi}{4}\right) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

in 2nd quadrant $\rightsquigarrow$ Sin is ⊕

$\frac{3\pi}{4} = \pi - \frac{\pi}{4}$



- Use Sin & Cos symmetries to find the symmetries for tan.

For example : $\tan(-x) = \frac{\sin(-x)}{\cos(-x)} = \frac{-\sin x}{\cos x}$

$= -\tan x$.

Derivative

$$\boxed{\begin{aligned} y = \sin x &\Rightarrow y' = \cos x \\ y = \cos x &\Rightarrow y' = -\sin x \end{aligned}}$$

$$y = \tan x \Rightarrow y' = 1 + \tan^2 x = \frac{1}{\cos^2 x} = \sec^2 x$$

Recall : $\sec x = \frac{1}{\cos x}$

identity : $\sec^2 x - \tan^2 x = 1$

$\boxed{\text{Trig Equations}}$ → Unknown is the angle

↳ Sin, Cos, tan of an angle is given, we want to find the angle.

⊛ In the first cycle $= [0, 2\pi]$ there are more than one angle, since different angles can have equal sin, cos or tan.

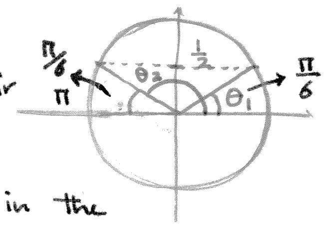
For example :

y-axis → $y = \frac{1}{2}$

$2\sin\theta = 1 \Rightarrow \sin\theta = \frac{1}{2}$

what is θ in $[0, 2\pi]$?

In the first round, two angles have their Sin equal to $\frac{1}{2}$.



By table ; the one in the 1st quadrant is $\frac{\pi}{6} \rightsquigarrow \theta_1 = \frac{\pi}{6}$ and from the unit circle you observe

$$\theta_2 = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

⊛ If we want the angles in more than one cycle, we should add the period of Sin to the angles we found above ⇝ 2nd round : $\pi/6 + 2\pi$

$5\pi/6 + 2\pi$

⇝ 3rd round : $\pi/6 + 4\pi$

$5\pi/6 + 4\pi$

⋮

In general solutions are :

$$\pi/6 + 2n\pi$$
$$5\pi/6 + 2n\pi \qquad n = 0, \pm 1, \pm 2, \dots$$

↓ multiples of 2π for repeating angles.

⊛ Sometimes, you have to simplify and factor terms before solving the equation.

Practice Problems.

1. Solve the following equations.

a) $\sin x + 2 = 3$

b) $3\tan^2 x = 1$

c) $2\cos^2\alpha - \sqrt{3}\cos\alpha = 0$

d) $9x^3\cos x - x\cos x = 0$

2. For each part :

I) Find the domain in $[0, 2\pi]$

II) Find the derivative at the given point.

a) $f(x) = \dfrac{\sin x}{\sqrt{3} - \tan x}, \quad x = 0$

b) $g(t) = \dfrac{1}{e^t(\cos t + 1)}, \quad t = \frac{\pi}{2}$

3. Find the solution to the following equations in $[0, 2\pi]$.

(You may use $\sin^2 x + \cos^2 x = 1$)

a) $\tan^2\theta + 3 = 0$

b) $\sin^2 x - \frac{1}{2} = \sin x - \cos^2 x$

c) $2\sin 2x = \sqrt{3}$

d) $4\sin x = 8$

e) $e^{4x}x^2\cos^2 x + x^2e^{4x}\sin^2 x = 0$

4. For each of the following angles

I) Find the quadrant at which the terminal side of the angles lies.

II) Find Sin and Cos of the angle.

III) Find tan and Sec of the angle.

a) $-\frac{\pi}{4}$:

b) $\frac{7\pi}{3}$:

c) $\frac{18\pi}{2}$:

d) $\frac{19\pi}{6}$:

e) $\frac{5\pi}{6}$:

f) $-\frac{2\pi}{3}$:

5. Find the following limits:

a) $\lim_{x \to 0} \frac{\cos x}{\sin x - 3} =$

b) $\lim_{x \to \pi} e^{x}(x + \cos x) =$

c) $\lim_{\theta \to \pi/4} \sin\left(\theta + \pi/4 \tan(\pi/2 - \theta)\right) =$

d) $\lim_{t \to 0^+} \frac{1}{\sin t} =$

e) $\lim_{x \to 3\pi/2^-} \tan x =$

6. Use IVT to determine if the graph of the curve $y = \frac{1}{\pi} x \tan x$ crosses the line $y = \frac{1}{8}$ at some point in $[0, \pi]$. Justify your answer.

7. differentiate each of the following:

a) $f(x) = 3/x^4 - x^2 \sin x$

b) $g(r) = 5e^{r}\cos r + \tan r$

c) $h(t) = \frac{\sin t}{3 - 2\cos t}$

d) $u(x) = x^3 e^x \tan x$

8. Find the equation of the line tangent to the graph of $f(x) = 2x^3 + 3\cos x - e^x$ at $x = 0$.

9. Suppose $g(\theta) = \frac{1}{\sin\theta + \frac{\theta}{2}}$

a) Find θ such that g has a horizontal tangent line?

b) Find θ at which the tangent line is parallel to the line $2y - x = 6$.

10. If $R(t) = \frac{\sin x}{x \cos x}$

Find the instantaneous rate of change in R, when $t = \pi$.