Practice.

Draw & write a formula for the function described:

(1) f is cont's at $x=1$, but discont's at $x=2$ because

$$\lim_{x \to 2^+} f(x) \neq \lim_{x \to 2^-} f(x)$$

(2) f is discont's at $x=2$, because $f(2)$ is NOT defined,

But $\lim_{x \to 2} f(x)$ exists.

(3) f is discont's at $x = 2$

because

$$\lim_{x \to 2^-} f(x) = +\infty$$

$$\lim_{x \to 2^+} f(x) = -\infty$$

Solution :

Cont's at $x=1$: $\lim$ exists when $x \to 1$ & $f(1)$ defined } are equal

Dis cont's at $x=2$: 1) $\lim$ DNE { • right $\lim \neq$ left $\lim$ • ∞ $\lim$

2) $f(2)$ NOT defined $\to$ Not in the domain

for example : open dot at $x=2$

3) (1) and (2) are OK but NOT equal.

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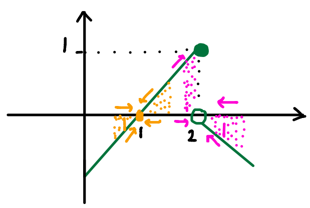
(1)



$f(1) = 0$

$\lim\limits_{x \to 1} f(x) = 0$ $\Longrightarrow$ f cont's at $x=1$

$f(2) = 1$

$\lim\limits_{x \to 2^-} f(x) = 1$

$\neq$ $\Longrightarrow$ f discont's at $x=2$

$\lim\limits_{x \to 2^+} f(x) = 0$

Not a unique answer!
Since the left and right $\lim$ are NOT equal the function must have a jump at $x=2$.

To write an equation with jump we can use piecewise functions such that when $x \to 2^-$ $(x<2)$ plug in 2 gives us 1

so $\lim\limits_{x \to 2^-} f(x) = 1$

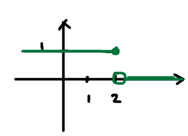
and when $x \to 2^+$ $(x>2)$ plug in 2 returns 0

so $\lim\limits_{x \to 2^+} f(x) = 0$

and when $x=2$, $f(2)=1$ $(x \leq 2)$

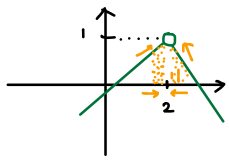
$$f(x) = \begin{cases} x-1 & x \leq 2 \\ -x+2 & x > 2 \end{cases}$$

or any piecewise function for which the right & left limits are NOT equal. For example : $f(x) = \begin{cases} 1 & x \leq 2 \\ 0 & x > 2 \end{cases}$



(2) lim exists $\rightsquigarrow$ left lim = right lim

But $f(2)$ NOT defined $\rightsquigarrow$ open dot at $x=2$

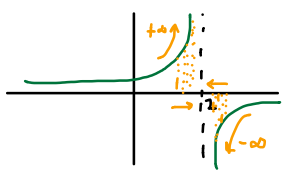


$$f(x) = \begin{cases} x-1 & x<2 \\ -x+1 & x>2 \end{cases}$$

$\longrightarrow$ NO $\leq$ or $\geq$ because $f(2)$ NOT defined

(3) $x \to 2$ we have $\pm\infty$ lims : In the function equation when we plug in $x=2$ we should get $\frac{\text{number}}{0}$

Thus we must have $(x-2)$ in the denominator



Negative to get the right sign for ∞.

$$f(x) = \frac{-4}{x-2}$$

can be any negative number.

$$\Rightarrow \lim_{x\to 2^-} f(x) = \frac{-4}{2^- - 2} = \frac{-4}{0^-} = +\infty$$

$$\lim_{x\to 2^+} f(x) = \frac{-4}{2^+ - 2} = \frac{-4}{0^+} = -\infty$$

⊛ Any equation that gives $\frac{\text{number}}{0}$ works.

to get $+\infty$ when $x \to 2^-$: $\frac{\ominus \to \text{negative number}}{x-2} = \frac{-}{0^-} = +\infty$

$-\infty$ 〃 $x \to 2^+$: $\frac{\ominus \to \text{negative}}{x-2} = \frac{-}{0^+} = -\infty$

⊛ You can play with the equation and make the desired limits happen, this is just one example.