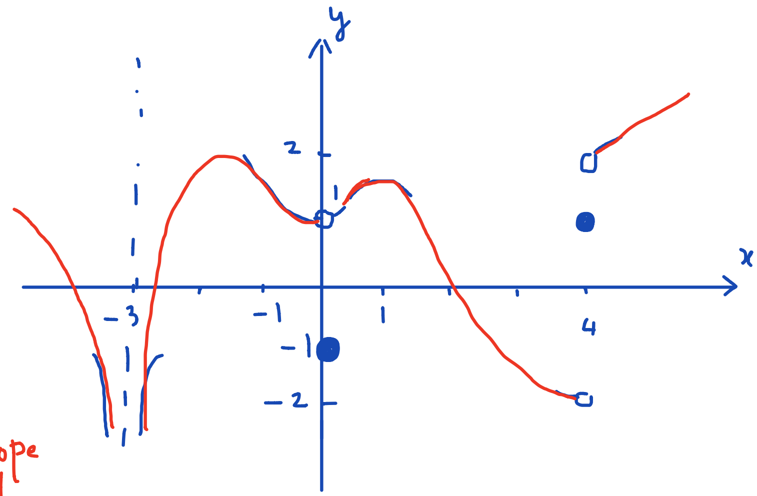
Homework 3, MATH 110-003

Due date: Tuesday, Oct 25, 2016 (in class)

Hand in full solutions to the questions below. Make sure you justify all your work and include complete arguments and explanations. Your answers must be clear and neatly written, as well as legible (no tiny drawings or micro-handwriting please!). ***Your answers must be stapled, with your name and student number at the top of each page.***

1. Sketch the graph of a function f which satisfies the following conditions.

- $\lim_{x\to 0} f(x) = 1$
- $\lim_{x\to 4^-} f(x) = -2$
- $\lim_{x\to 4^+} f(x) = 2$
- $\lim_{x\to -3} f(x) = -\infty$
- $f(0) = -1$
- $f(4) = 1$
- $f'(1) = 0$ → 0 slope
- $f'(-1) < 0$ negative slope at $x=-1$



Make sure you label the axes and clearly identify the points on the graph that are related to the above conditions. *Note: You are NOT required to provide a formula for $f(x)$.*

2. Show that there is a point in the interval $[1, 4]$ at which the two functions $f(x) = x - 1$, and $g(x) = \sqrt{x}$ intersect. (You must use IVT.)

3. Are there any points at which the function $f(x) = 2x^3 - \dfrac{3}{2}x^2$ has a horizontal tangent line? If yes, justify with your work.

4. If $f(x) = \dfrac{1}{\sqrt[5]{x^2}}$, then the slope of the perpendicular line to the function f at $x = 1$ is:

a) 1 b) $-\dfrac{2}{5}$ c) $\dfrac{5}{2}$ d) $-\dfrac{7}{5}$

② $f(x) = x - 1$
$g(x) = \sqrt{x}$ $\xrightarrow[\text{intersection}]{}$ $f(x) = g(x)$
$[1,4]$

$$x - 1 = \sqrt{x}$$

$$h(x) = x - 1 - \sqrt{x} = 0$$

We should show that $h(x)$ has a root in the interval $[1,4]$.

Check the assumption first:

$h(x)$ is cont's in $[1,4]$. No need to worry about $\sqrt{x}$, because its domain of continuity is $[0,+\infty)$ which contains $[1,4]$.

Moreover, $h(1) = 1 - 1 - \sqrt{1} = -1$
$h(4) = 4 - 1 - \sqrt{4} = 1$] sign change

By IVT, there is a root in the interval $(1,4)$ i.e. there is a number c between 1 and 4 such that $h(c) = 0$

$$\Rightarrow g(c) = f(c)$$

$\Rightarrow$ there is a point of intersection.

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3) $f(x) = 2x^3 - \frac{3}{2}x^2$

Horizontal tangent line $\rightsquigarrow m_{tan} = 0 \rightsquigarrow f'(x) = 0$ ⊛

So we want to find x, such that $f'(x) = 0$

From Power Rule: $f'(x) = 2(3x^2) - \frac{3}{2}(2x) = 6x^2 - 3x$

From ⊛ $6x^2 - 3x = 0$

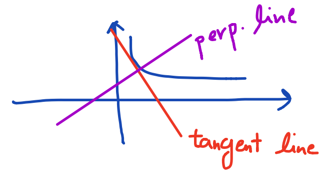
$\xrightarrow{\text{solve for } x}$ $3x(2x-1) = 0 \Rightarrow x = 0, \; x = \frac{1}{2}$

plug into f
to find
y-coordinates

$y = 0$, $y = 2\left(\frac{1}{2}\right)^3 - \frac{3}{2}\left(\frac{1}{2}\right)^2$
$= \frac{1}{4} - \frac{3}{8} = \frac{-1}{8}$

So points: $\boxed{(0,0)}$ and $\boxed{\left(\frac{1}{2}, -\frac{1}{8}\right)}$

4) $f(x) = \dfrac{1}{\sqrt[5]{x^2}}$



for two perpendicular lines :

$$m_{tan} = -\frac{1}{m_{per}}$$

We also know $m_{tan} = f'(1)$

$$f(x) = x^{-\frac{2}{5}} \Rightarrow f'(x) = -\frac{2}{5} x^{-\frac{2}{5}-1} = -\frac{2}{5\sqrt[5]{x^7}}$$

$$\Rightarrow f'(1) = -\frac{2}{5\sqrt[5]{1}} = -\frac{2}{5}$$

$$\Rightarrow m_{tan} = -\frac{2}{5} \quad \Rightarrow \boxed{m_{per} = +\frac{5}{2}}$$

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