

Homework 4, MATH 110-003
Due date: Tuesday, Nov 8, 2016 (in class)

Hand in full solutions to the questions below. Make sure you justify all your work and include complete arguments and explanations. Your answers must be clear and neatly written, as well as legible (no tiny drawings or micro-handwriting please!). Your answers must be stapled, with your name and student number at the top of each page.

Question 1. The functions f and g are given by the following graphs.

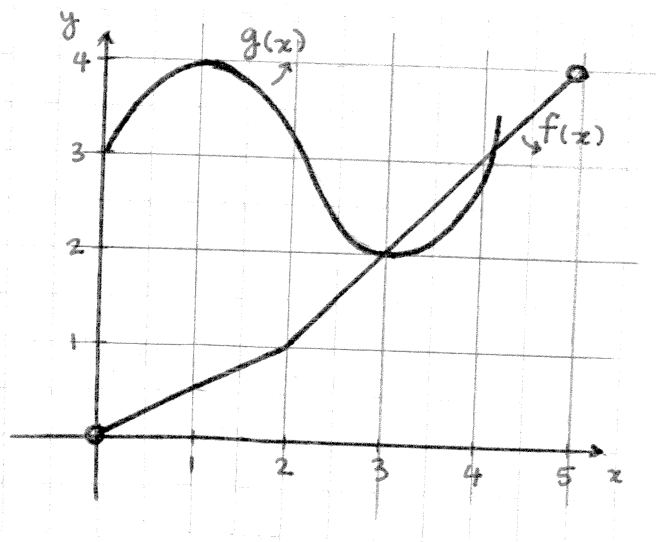
Find the following values:

a) Domain of f'

b) Roots of g'

c) $(fg)'(3)$

d) $\left(\frac{f}{g}\right)'(1)$



Question 2. On what intervals is the function $f(x) = x^3e^x$ increasing?

Question 3. Suppose

$$f(t) = \begin{cases} e^t \sin t & t < k \\ e^t \cos t & t > k \end{cases}$$

a) For what values of k does $\lim_{t \rightarrow k} f(t)$ exist?

b) Show that $f(t)$ is NOT continuous at the smallest k you found in part (a)?

c) Add a line to the function f such that it becomes continuous at the smallest k you found in part (a), and call this new function $g(t)$?

d) Is $g(t)$ differentiable at at the smallest k you found in part (a)?

Question 4. The function $f(x)$ is given by

$$f(x) = \frac{2 \sin x}{e^x(1 + \cos x)}$$

- a) Find the domain of f . (Interval notation is NOT required.)
- b) Find the derivative of f .
- c) Find the equation of the tangent line to $f(x)$ at $x = 0$.

Question 5. Use product rule to prove that

$$\frac{d}{dx}(f(x))^3 = 3(f(x))^2 f'(x)$$

(Hint: By using product rule first show that $(fgh)' = f'gh + fg'h + fgh'$, and then take $f = g = h$.)

(Recall that $\frac{d}{dx}$ is another notation for the derivative.)