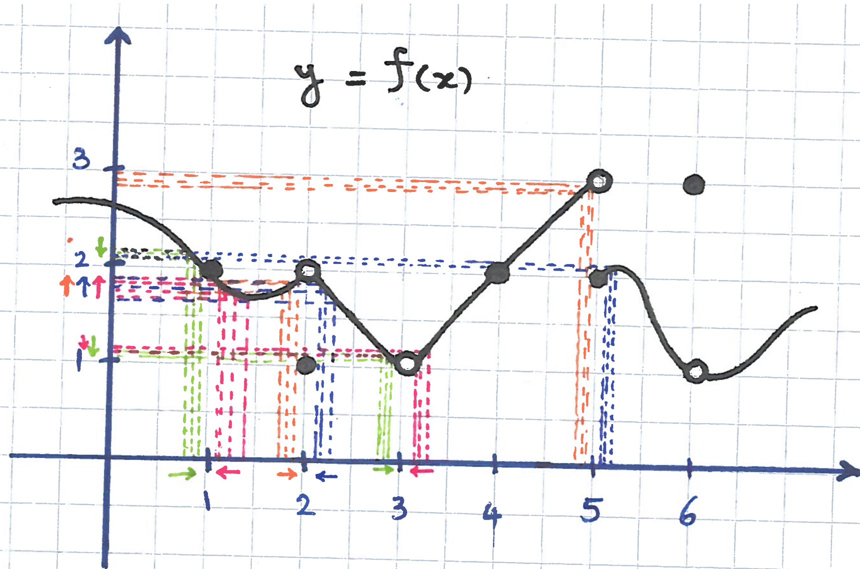



$$\lim_{x \to 1} f(x) = 2$$

* As $x$ approaches 1 (NOT equal to 1)
$f(x)$ approaches 2.

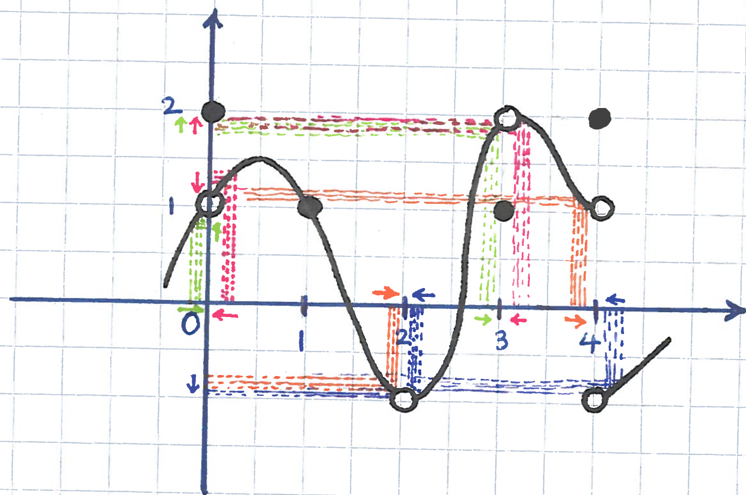
$$\lim_{x \to 2} f(x) = 2$$ However, $f(2) = 1$

$$\lim_{x \to 3} f(x) = 1$$ Even though,

$f(3)$ is NOT defined at 1.

$$\lim_{x \to 5} f(x) = \text{Does NOT Exist} = \text{DNE}$$

Because As $x$ approach 5 <u>from right</u>
$f(x)$ approaches 2
and <u>from left</u>, $f(x)$ approaches 3.




$$\lim_{x \to 0} f(x) = 1$$

However,

$f(0) = 2$

$$\lim_{x \to 2} f(x) = -1$$

$f(2)$ is NOT defined.

$$\lim_{x \to 3} f(x) = 2$$

But $f(3) = 1$

$$\lim_{x \to 4} f(x) = \text{DNE}$$

from left : 1

from right : $-1$

and $f(4) = 2$

$$f(t) = -10t^2 + 40t + 100$$

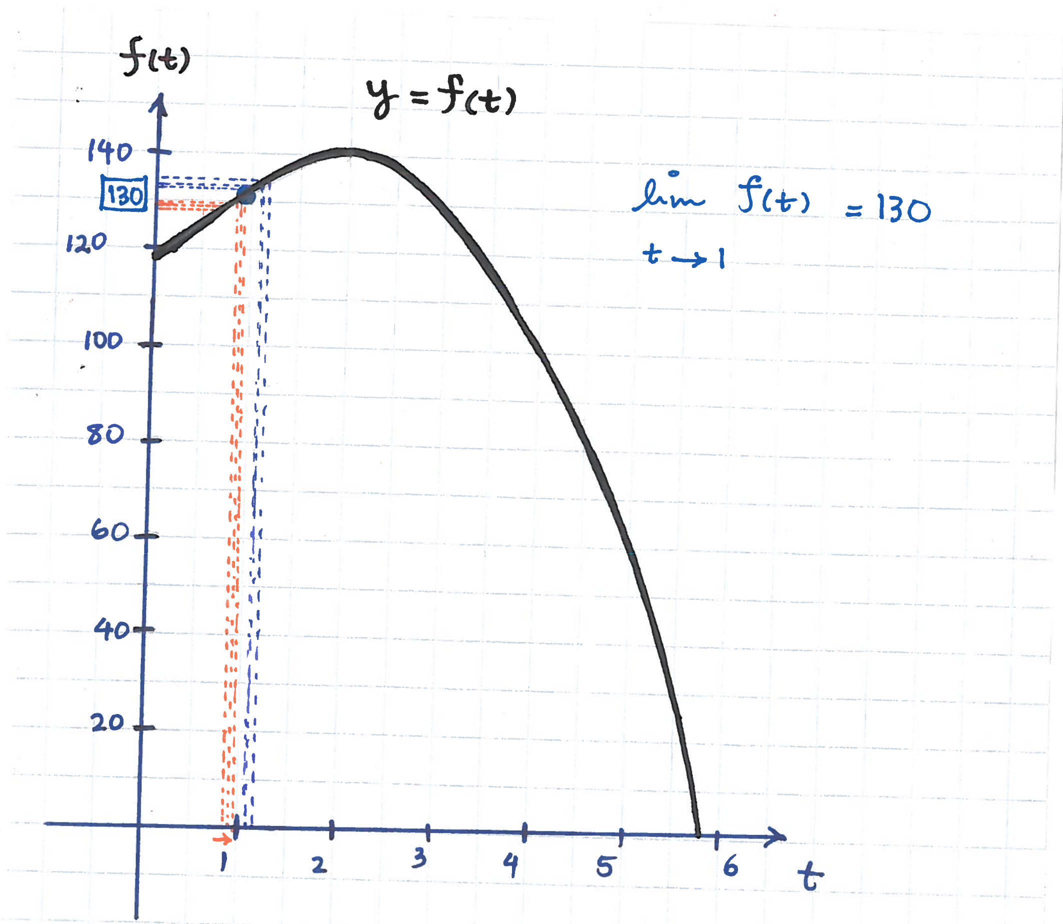
$$\lim_{t \to 1} f(t) = 130$$

| $t$ | $f(t)$ |
|---|---|
| 0.85 | 126.775 |
| 0.95 | 128.975 |
| 0.99 | 129.799 |
| 0.999 | 129.9799 |
| 0.9999 | 129.99799 |
| ↓ 1 | ↓ 130 |

→ Table of Values:

As $t$ approaches 1 from left and right, $f(t)$ approaches 130.

| $t$ | $f(t)$ |
|---|---|
| 1.15 | 132.775 |
| 1.05 | 130.975 |
| 1.01 | 130.199 |
| 1.001 | 130.0199 |
| 1.0001 | 130.00199 |
| ↓ 1 | ↓ 130 |




$$\lim_{t \to 1} f(t) = 130$$

**Example.** Find (Using table of values)

$$\lim_{r \to 3} \frac{2r^2 - r - 1}{r - 1} = 7$$

$$\lim_{r \to 1} \frac{2r^2 - r - 1}{r - 1} = 3$$

$\boxed{r \to 3}$

| $r$ | $f(r)$ |
|---|---|
| 2.9 | 6.8 |
| 2.99 | 6.98 |
| 2.999 | 6.998 |
| $\downarrow$ | $\downarrow$ |
| 3 | 7 |

| $r$ | $f(r)$ |
|---|---|
| 3.1 | 7.2 |
| 3.01 | 7.02 |
| 3.001 | 7.002 |
| $\downarrow$ | $\downarrow$ |
| 3 | 7 |

$\boxed{r \to 1}$

| $r$ | $f(r)$ |
|---|---|
| 0.9 | 2.8 |
| 0.99 | 2.98 |
| 0.999 | 2.998 |
| $\downarrow$ | $\downarrow$ |
| 1 | 3 |

| $r$ | $f(r)$ |
|---|---|
| 1.1 | 3.2 |
| 1.01 | 3.02 |
| 1.001 | 3.002 |
| $\downarrow$ | $\downarrow$ |
| 1 | 3 |

$$\lim_{x \to a} f(x) = L$$

Independent variable

Numbers

**Meaning:** As the independent variable $x$ approaches/gets close to the number $a$ (But NOT equal to $a$), then $f(x)$ gets close to the number $L$.

The letters to denote variables, functions and numbers can vary. For example:

- $\lim_{t \to a} g(t) = L$

  $g$ function of "t"

  indep var

  number

- $\lim_{r \to c} f(r) = M$

  $f$ function of "r"

  indep var

  number

- $\lim_{b \to j} u(b) = K$

  $u$ function of "b"

  indep var

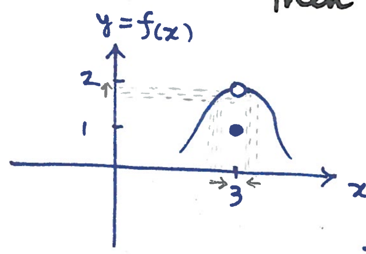
  number

# Q. True / False

( Justify or give a counter-example. )

**(a)** If $\lim_{x \to 3} f(x) = 2$ then $f(3) = 2$. F

Counter-example




$\lim_{x \to 3} f(x) = 2$

But $f(3) = 1 \neq 2$

In general :

$\lim_{x \to a} f(x) = L$ does NOT imply $f(a) = L$

**(b)** If $g(a) = 0$ then $\lim_{x \to a} \dfrac{f(x)}{g(x)}$ can NOT be computed. ( Not defined ) F

Counter-example $f(x) = 2x^2 - x - 1$

$g(x) = x - 1$

$a = 1$

$\Rightarrow \lim_{x \to 1} \dfrac{f(x)}{g(x)} = \lim_{x \to 1} \dfrac{2x^2 - x - 1}{x - 1} = 3$

But $g(a) = g(1) = 1 - 1 = 0$

Go to the example we did with table of values.

Even with calculators it is not easy to find a limit of a function with table of values or graphs are also not always easy to be sketched, so we'd like to find methods to determine the limit of a function algebraically by manipulating its equation. Go back to the example of the ball:

$f(t) = -10t^2 + 40t + 100$

From table of values & from the graph:

$$\lim_{t \to 1} f(t) = 130$$

We note that if we evaluate $f(t)$ at $t=1$ then

$$f(1) = 130$$

therefore;

$$\lim_{t \to 1} f(t) = f(1) = -10(1)^2 + 40 \times 1 + 100$$
$$= 130$$

And similarly:

$$\lim_{t \to 2} f(t) = -10(2)^2 + 40 \times 2 + 100$$
$$= -40 + 80 + 100 = 140$$

and assuming "t" is not time (any independent variable)

$$\lim_{t \to -1} f(t) = -10(-1)^2 + 40 \times (-1) + 100$$
$$= -10 - 40 + 100 = 50$$

• Algebraically find the limit :

In many cases, we can directly substitute $x$ and find the limit. Therefore, in algebraic method

First step : <u>Substitution</u>

Example :

• $\lim_{x \to 4} \sqrt{2x-5} = \sqrt{2 \times 4 - 5} = \sqrt{8-5} = \boxed{\sqrt{3}}$

• $\lim_{x \to -8} \sqrt[3]{x} + \sqrt{-x} + \frac{1}{x} = \sqrt[3]{-8} + \sqrt{-(-8)} + \frac{1}{-8}$

$= -2 + \sqrt{8} - \frac{1}{8}$

$= \frac{-16-1}{8} + \sqrt{8} = \boxed{\frac{-17}{8} + \sqrt{8}}$

• $\lim_{r \to 3} \frac{2r^2 - r - 1}{r-1} = \frac{2 \times (3)^2 - 3 - 1}{3-1} = \frac{18-4}{2} = \frac{14}{2} = \boxed{7}$

• $\lim_{r \to 1} \frac{2r^2 - r - 1}{r-1} = \frac{2 \times (1)^2 - 1 - 1}{1-1} = \frac{2-2}{1-1} = \frac{0}{0}$ !!!

However, from table of values we found that

$$\lim_{r \to 1} \frac{2r^2 - r - 1}{r-1} = 3$$