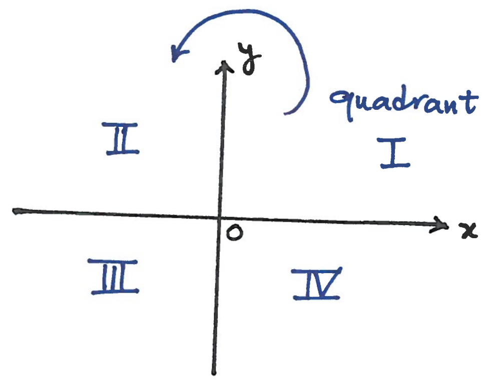
Lecture 2 : Lines in 2D plane

→ Cartesian (x-y) plane.

x-y plane consists of two perpendicular axes, which by convention, the horizontal axis is denoted by x and the vertical axis is denoted by y.

The two axes intersect at origin, O, which can be considered as the reference (starting) point.



→ x and y axes divide the plane into 4 regions, which are called quadrants.

* How to label the quadrants ?

We start from the top quadrant on the right, and we move counter - clockwise.

* Signs. Depending on which quadrant a point lies on, we have different signs for x and y.

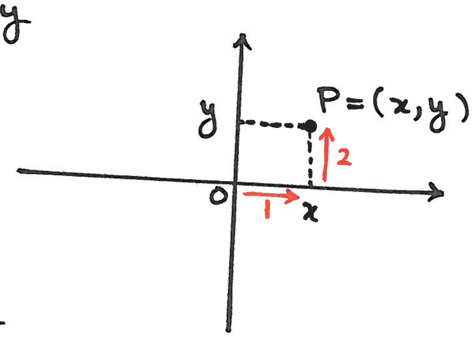
q \ sign	x	y
I	+	+
II	−	+
III	−	−
IV	+	−

→ Cartesian coordinate system.

Each point P on the x-y plane can be identified by two numbers denoted by

$P = (x, y)$

It means : Always start from O first move x-units horizontally to the left or right and then



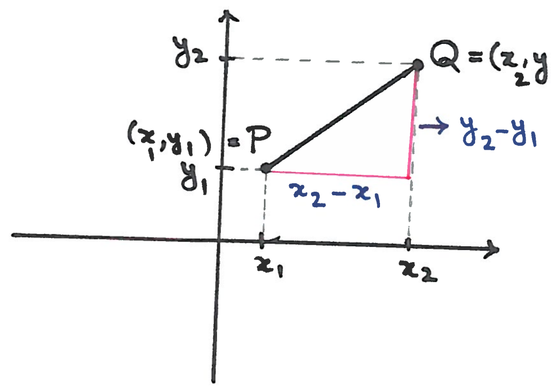
move y-units to up or down until you reach P.

- Move to the right / upward ⇝ positive direction
- Move to the left / downward ⇝ negative direction.

Distance between points.

Suppose $P=(x_1,y_1)$ and $Q=(x_2,y_2)$ are two points with given coordinates. We want to find the distance between P and Q, which is the length of the line segment, and we denote it by $|PQ|$.

length

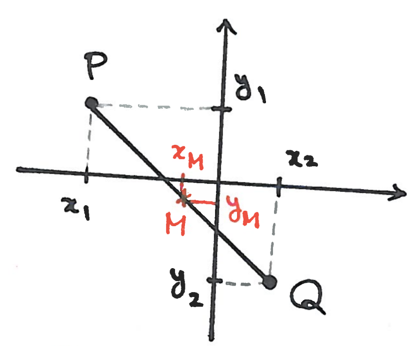


By Pythagoras formula :

$$|PQ| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Midpoint.

To find the coordinates of Midpoint M that lies in the middle of PQ, we just need to find the midpoint of two x-coordinates and y-coordinates.



$$M=(x_M,y_M)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

Ex 1. a) Find the distance between $R=(-1,3)$, $Q=(4,-4)$

$|RQ| = \sqrt{(4-(-1))^2+(-4-3)^2} = \sqrt{5^2+(-7)^2} = \sqrt{25+49} = \sqrt{74}$

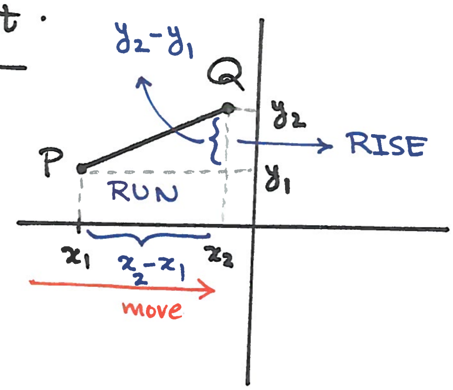
b) What is the midpoint, M, coordinates.

$M=\left(\frac{-1+4}{2},\frac{3+(-4)}{2}\right)=\left(\frac{3}{2},-\frac{1}{2}\right)$

Slope.

→ Slope of a line is a quantity that measures how fast the line is rising or falling moving from <u>left to right</u>.

We saw in our first class, given two points $P=(x_1,y_1)$ and $Q=(x_2,y_2)$ the slope of the line segment PQ (secant line) is given by



$$\text{Slope of } PQ = \frac{\text{RISE}}{\text{RUN}} = \frac{y_2-y_1}{x_2-x_1} = \frac{\Delta y}{\Delta x}$$

Notation. $\boxed{\text{slope} = m}$

$\boxed{\text{slope}}$: rate of change i
y-coordinates with respect t
rate of change in x-coordina

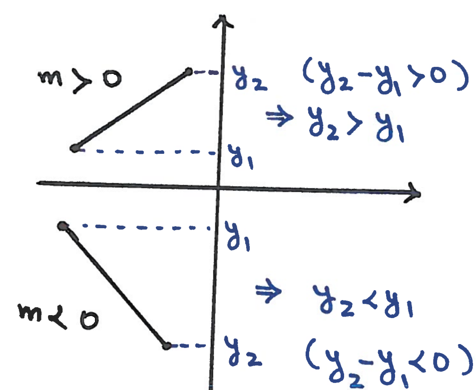
Ex 2. Find the slope of RQ in Ex 1.

$R=(\overset{x_1}{-1},\overset{y_1}{3})$

$Q=(\underset{x_2}{4},\underset{y_2}{-4})$ $\Rightarrow m = \dfrac{-4-3}{4-(-1)} = \dfrac{-7}{5} = -\frac{7}{5}$

positive slope ⇝ $y_2 > y_1$ ⇝ Rising line

negative slope ⇝ $y_2 < y_1$ ⇝ Falling line



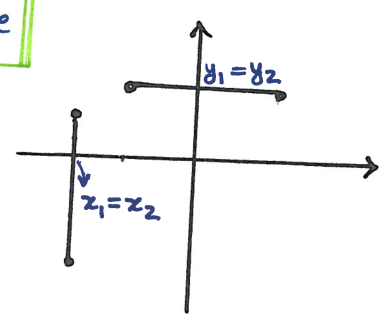
slope $\boxed{m = 0}$ ⇝ numerator $=0$

$\Rightarrow y_2 = y_1 \Rightarrow$ $\boxed{\text{Horizontal line}}$

$\boxed{m = \text{undefined}}$ ⇝ denominator $=0$

$\Rightarrow x_2 = x_1$

$\Rightarrow$ $\boxed{\text{Vertical line}}$



Equation of the line.

To find the equation of a line, we need two pieces of information:

1) slope of the line

2) a point on the line

Depending on the information we are given, we may call the equation one of the following:

point-slope eqt

$P = (x_1, y_1)$ $\quad m$

Go directly to the equation

2-point eqt

$P = (x_1, y_1)$

$Q = (x_2, y_2)$

First find the slope by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

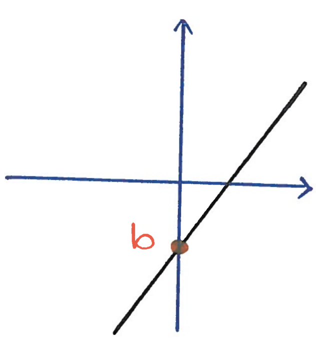
Choose one of the points. (say P)

eqt of the line

$$\boxed{y - y_1 = m(x - x_1)}$$

After simplification, we can write the formula with y at one side, and the rest at the other side of the eqt.
This form gives us

Slope-intercept eqt



$$y = mx + b$$

slope

y-intercept : where the line intersects y-axis.

* It means the point $(0, b)$ is on the line.

Ex 3. What is the equation of the line through $(-1,1)$ and $(1,-5)$.

2-point eqt $\Rightarrow$ First find the slope

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-1}{1-(-1)} = \frac{-6}{2} = -3$$

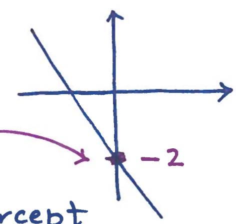
choose $(-1,1)$

$$\Rightarrow \; y - y_1 = m(x - x_1)$$

$$\Rightarrow \; y - 1 = -3(x - \overset{+1}{\cancel{(-1)}}) \xrightarrow{\text{simplify}} y - 1 = -3x - 3$$

$$\Rightarrow \; y = -3x - 2$$

Falling Slope=m y-intercept



Ex 4. What is the slope and y-intercept of the line

$$2x - 3y = 6$$

Simplify to make it of the form $y = mx + b$

$$-3y = 6 - 2x$$

$$\xrightarrow[-3]{\text{divide by}} \; y = -2 + \frac{2}{3}x$$

$\longrightarrow m = \frac{2}{3}$

$\longrightarrow$ y-intercept $= -2$

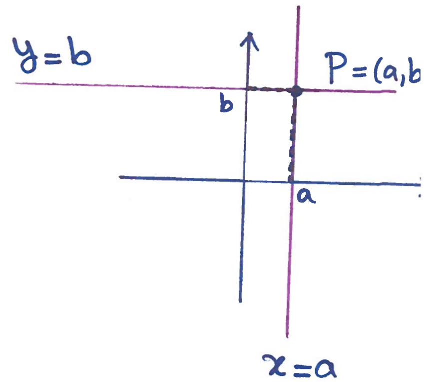
Horizontal & Vertical lines.

for a horizontal line : $m = 0$, so the equation $y = mx + b$ transforms to $y = b$

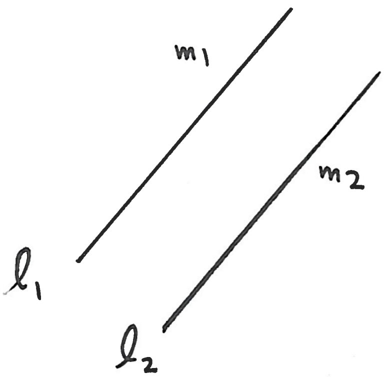
for a vertical line : m = undefined, however for a vertical line all the points have the same x, therefore

If $P = (a,b)$ is a given point then

- the horizontal line through $P \Rightarrow \boxed{y = b}$
- the vertical line through $P \Rightarrow \boxed{x = a}$

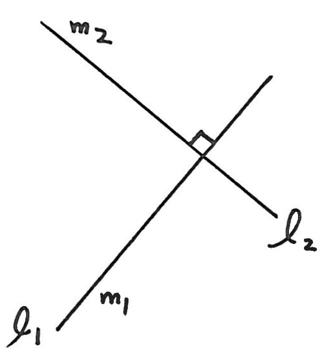


Parallel & Perpendicular lines.



two lines l_1 and l_2 with slopes m_1 and m_2 are parallel and denoted by $l_1 \parallel l_2$ when their slopes are equal.

$$\boxed{l_1 \parallel l_2 \iff m_1 = m_2}$$



l_1 and l_2 are perpendicular and denoted by $l_1 \perp l_2$ when their slope is negative and reciprocal.

$$\boxed{l_1 \perp l_2 \iff m_1 = -\frac{1}{m_2}}$$

Ex 5. Find the eqt. of the line perpendicular to the line in Ex 4 and with y-intercept $-\frac{1}{2}$.

We need to find the slope and we have the intercept, so we use $y = mx + b$.

$m_{Ex4} = \frac{2}{3} \Longrightarrow m_{Per} = -\frac{3}{2} \Rightarrow y = -\frac{3}{2}x - \frac{1}{2}$ → y-intercept