Name (print):

Student number:

Section *(Please circle one)*: 001 002 003 004

University of British Columbia

MATH 110: MIDTERM TEST 1

Date: *October 16, 2013*
Time: *6:00 p.m. to 7:30 p.m.*
Number of pages: *9 (including cover page)*
Exam type: *Closed book*
Aids: *No calculators or other electronic aids*

Rules governing formal examinations:

Each candidate must be prepared to produce, upon request, a UBC card for identification.

No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.

Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:

- *Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;*
- *Speaking or communicating with other candidates;*
- *Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.*

Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

For examiners' use only		
Question	Mark	Possible marks
1		7
2		4
3		5
4		6
5		4
6		8
7		6
Total		40

1. Let P be the point $(1, 2)$ and C be the circle described by the equation

$$(x-5)^2 + (y-5)^2 = 25.$$

All three parts of this question refer to the point P and the circle C.

2 marks (a) Does the point P lie on the circle C? Justify your answer.

2 marks (b) Find the equation of the line that passes through the point P and the centre of the circle C.

3 marks (c) The line whose equation you found in part (b) intersects the circle C at exactly two points. Find the coordinates of those points. *Hint: Draw a picture.*

2. Both parts of this question refer to the functions

$$f(x) = \sqrt{2x+1} \text{ and } g(x) = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}.$$

2 marks (a) On its entire domain, the function $(f \circ g)(x)$ has a *range*, or output, of exactly two numbers. Find those numbers, and explain your answers.

2 marks (b) On its entire domain, the function $(g \circ f)(x)$ also has a range of exactly two numbers. Find those numbers, and explain your answers.

3. Suppose the position of a particle with respect to time is described by the function

$$p(t) = 2t^2 - 3t + 4,$$

where position is measured in metres and time is measured in seconds. All three parts of this question refer to this particle.

2 marks (a) Calculate the average velocity of the particle between $t = 2$ seconds and $t = 3$ seconds.

2 marks (b) Calculate the average velocity of the particle between $t = 2$ seconds and $t = 2 + h$ seconds, where h is a small positive number.

1 mark (c) Estimate the instantaneous velocity of the particle at $t = 2$ seconds, and explain your answer.

4. Evaluate each of the following limits or show that the limit does not exist.

2 marks (a) $\lim_{x \to 4} 2x^2 - 5x - 12$

2 marks (b) $\lim_{x \to 4} \dfrac{2x^2 - 5x - 12}{x - 4}$

2 marks (c) $\lim_{x \to 4} \dfrac{2x^2 - 5x - 12}{|x - 4|}$

4 marks 5. Show that the function $f(x) = x^3 - 3x^2 + 1$ has at least three roots in the interval $[-1, 3]$.

[1 bonus mark] Explain why $f(x)$ cannot have more than three roots in the interval $[-1, 3]$.

6. Determine whether each of the following statements is true or false. If it is true, provide justification. If it is false, provide a counterexample.

2 marks (a) The graph of $f(x) = \frac{1}{2}\left((x+3)^2 - 2\right)$ crosses the x-axis.

2 marks (b) If $\lim_{x \to 1} f(x) = 4$, then $f(1) = 4$.

2 marks (c) If $f\left(\frac{1}{10}\right) = 2$, $f\left(\frac{1}{100}\right) = 2$, $f\left(\frac{1}{1000}\right) = 2$, and in fact $f\left(\frac{1}{10^n}\right) = 2$ for every integer n, then $f(0) = 2$.

2 marks (d) $f(x) = \begin{cases} \frac{3}{x+2} & \text{if } x < 1 \\ \sqrt{x} & \text{if } x \geq 1 \end{cases}$ is continuous at all real numbers.

6 marks 7. On the grid below, sketch a function $f(x)$ satisfying all of the following 6 properties:

- The domain is $[-10, -6) \cup (-6, 10]$.
- For every value x in the domain, $-4 \leq f(x) \leq 6$.
- $\lim_{x \to 6} f(x) = 4$
- $f(6) = 0$
- $\lim_{x \to -2^-} f(x) = 2$
- $\lim_{x \to -2^+} f(x) = -2$

You are not required to come up with an algebraic equation for the function.

This page may be used for rough work. It will not be marked.