Practice. Find the LCD of the following expressions and simplify them.

a) $\dfrac{x}{x-1} + \dfrac{2}{(x-1)^2} =$

b) $\dfrac{x+1}{x^2(x-1)} - \dfrac{5}{x(x-1)^2} =$

c) $\dfrac{\dfrac{1}{x} - \dfrac{1}{x^2}}{x + \dfrac{1}{x-2}} =$

a) $(x-1) \not\times (x-1)^2 \rightsquigarrow$ LCD : $(x-1)^2$

$$\frac{x\cdot(x-1)}{x-1\cdot(x-1)} + \frac{2}{(x-1)^2} = \frac{x(x-1)+2}{(x-1)^2} = \frac{x^2-x+2}{(x-1)^2}$$

($x-1$) missing; NO change; NO simpler! No factors.

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b) $x^2 \left[\not\times \right] (x-1)^2 \rightsquigarrow$ LCD : $x^2(x-1)^2$

$\not\times$: $x(x-1)^2$

$$\frac{x+1\cdot(x-1)}{x^2(x-1)\cdot(x-1)} - \frac{5\cdot x}{x(x-1)^2\cdot x} = \frac{(x+1)(x-1) - 5x}{x^2(x-1)^2}$$

($x-1$) missing; x missing

$$= \frac{x^2 - 1 - 5x}{x^2(x-1)^2}$$

c) Top: $\frac{1\cdot x}{x\cdot x} - \frac{1}{x^2} = \frac{x-1}{x^2}$

LCD: x^2

Bottom: $x + \frac{(x-2)\ 1}{1\cdot(x-2)\ x-2} = \frac{x(x-2)+1}{x-2} = \frac{x^2-2x+1}{(x-2)} = \frac{(x-1)^2}{x-2}$

LCD: $x-2$

Practice.

a. $\lim_{x\to -1} \frac{-5}{x(x+1)^2} =$

b. $\lim_{x\to -1} \frac{-5}{x(x+1)} =$

c. $\lim_{x\to 1} \frac{-5}{x(x+1)} =$

a) $\lim_{x\to -1} \frac{-5}{x(x+1)^2} \overset{\text{sub}}{=} \frac{-5}{-1(-1+1)^2} = \frac{5}{(0)^2}$

Approach (-1) from left and right: $(x+1)^2 = (-1+1)^2 = 0^+$ (always positive)

$= \frac{5}{0^+} = +\infty$ But limit DNE

b) $\lim_{x\to -1} \frac{-5}{x(x+1)} = \frac{-5}{-1(-1+1)} = \frac{5}{0}$

No squaring ⇝ split into right & left limits:

$$\lim_{x \to -1^+} \frac{-5}{x(x+1)} = \frac{-5}{-1(-1^+ + 1)} = \frac{5}{0^+} = +\infty$$

limit DNE

$$\lim_{x \to -1^-} \frac{-5}{x(x+1)} = \frac{-5}{-1(-1^- + 1)} = \frac{5}{0^-} = -\infty$$

c) $$\lim_{x \to 1} \frac{-5}{x(x+1)} = \frac{-5}{1(1+1)} = \frac{-5}{2}$$

"Good" case! Substitution works!