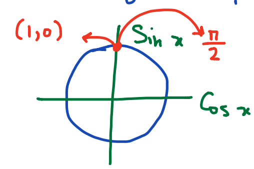
1) a) $\sin x + 2 = 3 \Rightarrow \sin x = 3 - 2 = 1$

$\Rightarrow \sin x = 1 \longrightarrow$ Go to unit circle, $\sin x$ is the y-axis. We should find the angle corresponding to $y = 1$.

$\Rightarrow x = \frac{\pi}{2}$



other cycles $\Rightarrow$ $\boxed{x = \frac{\pi}{2} + 2n\pi}$

$n = 0, \pm 1, \pm 2, \dots$

b) $3\tan^2 x = 1 \Rightarrow \tan^2 x = \frac{1}{3}$

Notation: $(\tan x)^2 = \frac{1}{3} \Rightarrow \begin{cases} \tan x = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}} & (\text{I}) \\ \tan x = -\sqrt{\frac{1}{3}} = -\frac{1}{\sqrt{3}} & (\text{II}) \end{cases}$

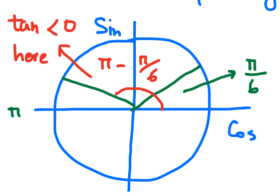
We must solve two equations I and II.

(I): $\tan x = \frac{1}{\sqrt{3}} \rightsquigarrow$ what angle has its $\tan = \frac{1}{\sqrt{3}}$?

$x = \frac{\pi}{6} \xrightarrow[\text{itself every } \pi]{\tan \text{ repeats}} \boxed{x = \frac{\pi}{6} + n\pi}$ other cycles

$n = 0, \pm 1, \pm 2, \dots$

(II): $\tan x = -\frac{1}{\sqrt{3}}$



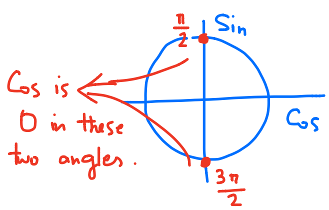
$x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\xrightarrow[\text{cycles}]{\text{other}} \boxed{x = \frac{5\pi}{6} + n\pi}$ $n = 0, \pm 1, \pm 2, \dots$

c) $2\cos^2\alpha - \sqrt{3}\cos\alpha = 0$

$\Rightarrow \cos\alpha\,(2\cos\alpha - \sqrt{3}) = 0 \Rightarrow \begin{cases} \cos\alpha = 0 & (\text{I}) \\ 2\cos\alpha - \sqrt{3} = 0 & (\text{II}) \end{cases}$

(I): $\cos\alpha = 0$

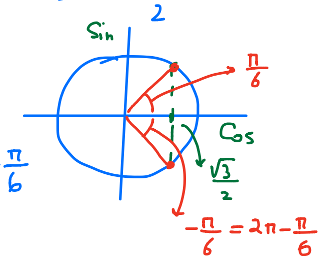


$\alpha = \frac{\pi}{2}$, $\alpha = \frac{3\pi}{2}$

Cos repeats itself every 2π

$\boxed{\begin{array}{l} \alpha = \frac{\pi}{2} + 2n\pi \\ \alpha = 3\pi/2 + 2n\pi \end{array}}$, $n = 0, \pm 1, \pm 2, \dots$

(II) $\cos\alpha = \frac{\sqrt{3}}{2}$



$\alpha = \frac{\pi}{6}$, $\alpha = -\frac{\pi}{6}$

other angles

$\boxed{\alpha = \frac{\pi}{6} + 2n\pi \ , \ \alpha = -\frac{\pi}{6} + 2n\pi \quad n = 0, \pm 1, \pm 2, \dots}$

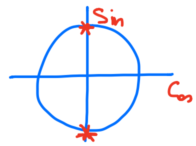
d) $9x^3\cos x - x\cos x = 0$

$\xrightarrow{\text{factor}}$ $x\cos x(9x^2-1)=0$

$\boxed{x=0}$

or $9x^2-1=0 \Rightarrow 9x^2=1 \Rightarrow x^2=\frac{1}{9} \Rightarrow \boxed{x=\pm\sqrt{\frac{1}{9}}=\pm\frac{1}{3}}$

or $\cos x = 0 \Rightarrow x=\frac{\pi}{2}, \frac{3\pi}{2}$



$\underset{\text{other cycles}}{\Rightarrow}$ $\boxed{x=\frac{\pi}{2}+2n\pi, \quad x=\frac{3\pi}{2}+2n\pi \qquad n=0,\pm1,\pm2,\dots}$

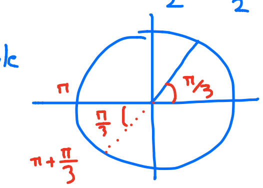
2) a) $f(x)=\dfrac{\sin x}{\sqrt{3}-\tan x}$ → everywhere OK

* the denominator must be Nonzero $\xrightarrow{\text{exclude}}$ $\sqrt{3}-\tan x = 0$
* also $\tan x$ is NOT ok everywhere $\xrightarrow{\tan=\frac{\sin}{\cos}}$ $\cos x = 0 \Rightarrow x=\frac{\pi}{2}, \frac{3\pi}{2}$

$\sqrt{3}-\tan x = 0 \Rightarrow \tan x = \sqrt{3} \Rightarrow$ Go to the memorized table

$+\tan \xrightarrow[\text{quadrant}]{1^{st}} x=\frac{\pi}{3}$

$\xrightarrow[\text{quad}]{3^{rd}} x=\pi+\frac{\pi}{3}=\frac{4\pi}{3}$



(I) $\Rightarrow$ Domain: Everywhere except $\{x=\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{4\pi}{3}\}$

or $x \neq \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{4\pi}{3}$

(II) $\xrightarrow[\text{Rule}]{\text{Quotient}}$ $f'(x)=\dfrac{\cos x(\sqrt{3}-\tan x)-(-\sec^2 x)}{(\sqrt{3}-\tan x)^2}$

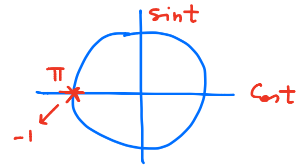
$\Rightarrow f'(0)=\dfrac{1(\sqrt{3}-0)+1}{(\sqrt{3}-0)^2}=\boxed{\dfrac{\sqrt{3}+1}{3}}$

b) $g(t)=\dfrac{1}{e^t(\cos t+1)}$ Find where the denominator is 0:

$e^t(\cos t + 1) = 0 \to e^t = 0 \to$ NO solution!

$\to \cos t + 1 = 0 \to \cos t = -1$

$\Rightarrow t = \pi$



(I)
$\Rightarrow$ Domain = everywhere except π

or $x \neq \pi$

product rule for the denominator

(II) $\to$ Rewrite: $g'(t) = \dfrac{0 - \left[e^t(\cos t + 1) + e^t(-\sin t)\right]}{\left(e^t(\cos t + 1)\right)^2}$

Quotient Rule

$$g'\left(\tfrac{\pi}{2}\right) = \frac{-\left(e^{\frac{\pi}{2}}(0+1) + e^{\frac{\pi}{2}}(-1)\right)}{\left(e^{\frac{\pi}{2}}(0+1)\right)^2} = \frac{-e^{\frac{\pi}{2}} + e^{\frac{\pi}{2}}}{e^{\pi}} = 0$$

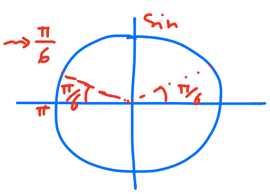
3) a) $\tan^2\theta = -3 \Rightarrow$ A squared value can NOT be equal to a negative number

$\Rightarrow$ NO solution!

b) $\sin^2 x - \frac{1}{2} + \cos^2 x = \sin x$

identity $\quad 1 - \frac{1}{2} = \sin x \Rightarrow \sin x = \frac{1}{2}$ table $\to \frac{\pi}{6}$

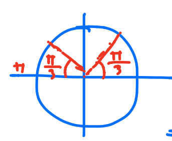
positive: 1st quad, 2nd



$\Rightarrow x = \frac{\pi}{6}$ and $x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

c) $\sin 2x = \frac{\sqrt{3}}{2}$ $\to$ table: $\frac{\pi}{3}$

positive: 1st quad $\quad 2x = \frac{\pi}{3}$



$\Rightarrow x = \dfrac{\frac{\pi}{3}}{2} = \frac{\pi}{6}$

2nd $\quad 2x = \pi - \frac{\pi}{3} = \frac{2\pi}{3} \Rightarrow x = \dfrac{\frac{2\pi}{3}}{2} = \frac{2\pi}{6} = \frac{\pi}{3}$

d) $\sin x = \frac{8}{4} = 2 \Rightarrow \sin x = 2$

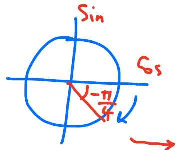
$\to$ Sin always returns a number between -1 and 1 (inclusive) $\Rightarrow$ No Solution!

e) $e^{4x}\, x^2 \left(\underbrace{\cos^2 x + \sin^2 x}_{1} \right) = 0$

$\Rightarrow e^{4x}\, x^2 = 0 \quad \Rightarrow \quad x^2 = 0 \quad \Rightarrow \underline{\underline{x = 0}}$

or $e^{4x} = 0 \Rightarrow$ NO solution!

4) a) $\underbrace{-}_{\text{negative direction}}\frac{\pi}{4}$ ⇝ clockwise

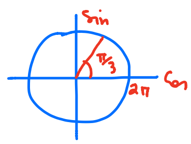


$\longrightarrow 4^{th}$ quadrant $\longrightarrow$ Sin $\ominus$ Cos $\oplus$ tan $\ominus$

table : $\sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$\cos\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ $\Rightarrow \tan\left(-\frac{\pi}{4}\right) = -1$ $\Rightarrow \sec\left(-\frac{\pi}{4}\right) = \frac{1}{\cos\left(-\frac{\pi}{4}\right)} = \frac{2}{\sqrt{2}} = \sqrt{2}$

b) $\frac{7\pi}{3} = \frac{6\pi+\pi}{3} = \underbrace{2\pi}_{\text{one complete cycle}} + \frac{\pi}{3}$ ← table

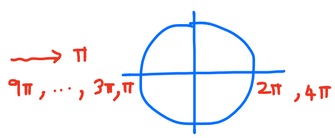


$\longrightarrow 1^{st}$ quadrant $\longrightarrow$ Sin $\oplus$ Cos $\oplus$ tan $\oplus$

table : $\sin\left(\frac{7\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$\cos\left(\frac{7\pi}{3}\right) = \frac{1}{2}$ $\Rightarrow \tan\left(\frac{7\pi}{3}\right) = \sqrt{3}$ $\Rightarrow \sec\left(\frac{7\pi}{3}\right) = \frac{1}{\frac{1}{2}} = 2$

c) $\frac{18\pi}{2} = 9\pi \longrightarrow \pi$



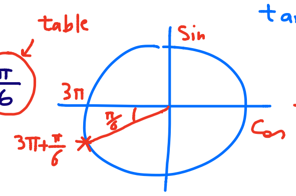
$\Rightarrow$ everything is like π:

$\sin(9\pi) = \sin\pi = 0$

$\cos(9\pi) = \cos\pi = -1$

$\tan(9\pi) = 0$, $\sec 9\pi = -1$

d) $\frac{19\pi}{6} = \frac{18\pi+\pi}{6} = \underbrace{3\pi}_{\text{equal to } \pi} + \frac{\pi}{6}$ ← table

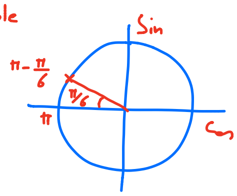


$\longrightarrow 3^{rd}$ quadrant Sin $\ominus$ Cos $\ominus$ $\Rightarrow$ tan $\oplus$

$\sin\left(\frac{19\pi}{6}\right) = -\frac{1}{2}$

$\cos\left(\frac{19\pi}{6}\right) = -\frac{\sqrt{3}}{2}$ $\Rightarrow \tan\left(\frac{19\pi}{6}\right) = \frac{1}{\sqrt{3}}$, $\sec\left(\frac{19\pi}{6}\right) = \frac{1}{-\frac{\sqrt{3}}{2}} = -\frac{2}{\sqrt{3}}$

e) $\frac{5\pi}{6} = \frac{6\pi-\pi}{6} = \pi - \frac{\pi}{6}$ ← table

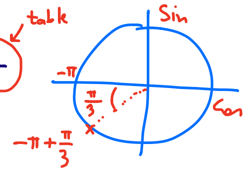


$\longrightarrow 2^{nd}$ quadrant $\hookrightarrow$ Sin $\oplus$ Cos $\ominus$ $\Rightarrow$ tan $\ominus$

$\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$

$\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$ $\Rightarrow \tan\left(\frac{5\pi}{6}\right) = -\frac{1}{\sqrt{3}}$, $\sec\left(\frac{5\pi}{6}\right) = -\frac{2}{\sqrt{3}}$

f) $-\frac{2\pi}{3} = \frac{-3\pi + \pi}{3} = -\pi + \frac{\pi}{3}$ ← talk



$\longrightarrow 3^{rd}$ quadrant

$\longrightarrow$ Sin $\ominus$, Cos $\ominus$ $\Rightarrow$ tan $\oplus$

$\sin\left(-\frac{2\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

$\cos\left(-\frac{2\pi}{3}\right) = -\frac{1}{2}$ $\Rightarrow \tan\left(-\frac{2\pi}{3}\right) = \sqrt{3}$, $\sec\left(-\frac{2\pi}{3}\right) = -2$

5) a) $\lim\limits_{x \to 0} \frac{\cos x}{\sin x - 3} \overset{\text{sub}}{=} \frac{\cos 0}{\sin 0 - 3} = \frac{1}{0-3} = -\frac{1}{3}$

b) $\lim\limits_{x \to \pi} e^{x}(x + \cos x) = e^{\pi}(\pi + \cos \pi) = e^{\pi}(\pi - 1)$

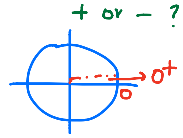
c) $\lim\limits_{\theta \to \frac{\pi}{4}} \sin\left(\theta + \frac{\pi}{4}\tan\left(\frac{\pi}{2} - \theta\right)\right) = \sin\left(\frac{\pi}{4} + \frac{\pi}{4}\tan\left(\underbrace{\cancel{\frac{\pi}{2} - \frac{\pi}{4}}}_{\frac{\pi}{4}}\right)\right) \overset{\tan\frac{\pi}{4} = 1}{=} \sin\left(\frac{\pi}{4} + \frac{\pi}{4}\right)$

$= \sin\frac{\pi}{2} = 1$

d) $\lim\limits_{t \to 0^+} \frac{1}{\sin t} = \frac{1}{\sin 0} = \frac{1}{0} = \frac{1}{0^+} = +\infty$

$t \to 0^+$ ↓ 1^{st} quadrant $\rightsquigarrow$ Sin $\oplus$

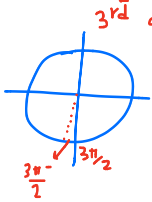
0 ↙ $+$ or $-$?



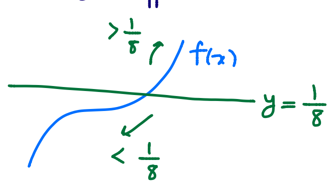
e) $\lim\limits_{x \to \frac{3\pi}{2}^-} \tan x = \tan\frac{3\pi}{2} =$ NOT defined

$= \frac{\sin\frac{3\pi}{2}}{\cos\frac{3\pi}{2}} = \frac{-1}{0^-} = +\infty$

$x \to \frac{3\pi}{2}^-$ ↓ 3^{rd} quadrant $\rightsquigarrow$ Cos $\ominus$



6) $y = \frac{1}{\pi} x \tan x$



in $[0, \pi]$ tan x is NOT defined at $x = \frac{\pi}{2}$ so we choose a sub interval like $[0, \frac{\pi}{4}]$ to avoid $\frac{\pi}{2}$.

Therefore $f(x)$ is Cont's in $[0, \frac{\pi}{4}]$ and

$f(0) = \frac{1}{\pi} 0 \tan 0 = 0 < \frac{1}{8}$

$f\left(\frac{\pi}{4}\right) = \frac{1}{\pi} \cdot \frac{\pi}{4} \tan\frac{\pi}{4} = \frac{1}{4} > \frac{1}{8}$ $\Rightarrow$ $\frac{1}{8}$ is a value between $f(0)$ and $f\left(\frac{\pi}{4}\right)$

$\Longrightarrow$ By IVT there is number c between 0 and $\frac{\pi}{4}$

such that $f(c) = \frac{1}{8} \Rightarrow$ crosses the line $y = \frac{1}{8}$.

7. a) $f(x) = 3x^{-4} - x^2 \sin x$

$\Rightarrow f'(x) = 3(-4x^{-4-1}) - (2x\sin x + x^2 \cos x) = -12x^{-5} - 2x\sin x - x^2\cos x$

b) $g(r) = 5e^r \cos r + \tan r$

$\Rightarrow g'(r) = 5\left(e^r \cos r + e^r(-\sin r)\right) + \sec^2 r$

c) $h(t) = \dfrac{\sin t}{3 - 2\cos t} \Rightarrow h'(t) = \dfrac{\cos t\,(3 - 2\cos t) - \sin t\,(-2(-\sin t))}{(3-2\cos t)^2}$

$= \dfrac{3\cos t - 2\cos^2 t - 2\sin^2 t}{(3-2\cos t)^2} = \dfrac{3\cos t - 2}{(3 - 2\cos t)^2}$

d) $u(x) = x^3 e^x \tan x$

$u'(x) = 3x^2 e^x \tan x + x^3 e^x \tan x + x^3 e^x \sec^2 x$

8) $f(x) = 2x^3 + 3\cos x - e^x$ at $x = 0$

$f(0) = 0 + 3 - 1 = 2 \Rightarrow (0,2)$ point

$f'(x) = 6x^2 - 3\sin x - e^x \Rightarrow f'(0) = 0 - 0 - 1 = -1 = m_{\tan}$

$\Rightarrow y - 2 = -1(x - 0) \Rightarrow \boxed{y = -x + 2}$

9) $g(\theta) = \dfrac{1}{\sin\theta + \frac{\theta}{2}} \longrightarrow$ Horizontal tangent line $g'(\theta) = 0$

$g(\theta) = \left(\sin\theta + \frac{\theta}{2}\right)^{-1} \Rightarrow g'(\theta) = -\left(\sin\theta + \frac{\theta}{2}\right)^{-2}\left(\cos\theta + \frac{1}{2}\right) = -\dfrac{\cos\theta + \frac{1}{2}}{\left(\sin\theta + \frac{\theta}{2}\right)^2}$

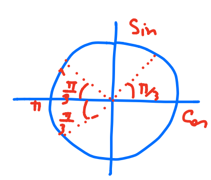
$g'(\theta) = 0 \xrightarrow[\text{numerator}=0]{\text{just}} \cos\theta + \frac{1}{2} = 0 \Rightarrow \cos\theta = -\frac{1}{2}$

quadrant — table

2nd and 3rd — $\frac{\pi}{3}$

$\Rightarrow \theta = \pi - \frac{\pi}{3} \Rightarrow \boxed{\theta = \frac{2\pi}{3}}$

$\theta = \pi + \frac{\pi}{3} \Rightarrow \boxed{\theta = \frac{4\pi}{3}}$



Ignore (b).

10. $R(t) = \dfrac{\sin t}{t \cos t}$ inst. rate of change at $t = \pi$ $\Rightarrow R'(\pi)$

$$R'(t) = \frac{\cos t\,(t \cos t) - \sin t\,(\cos t + t(-\sin t))}{(t \cos t)^2}$$

$$R'(\pi) = \frac{\cos \pi\,(\pi \cos \pi) - \overbrace{\sin \pi}^{0}\,(\cdots)}{(\pi \cos \pi)^2} = \frac{\pi}{\pi^2} = \boxed{\frac{1}{\pi}}$$