Worksheet Nov 15 : Logarithms

1. Evaluate the following logarithms.

a) $\log_2 16 =$

b) $\log_7 1 =$

c) $\ln \sqrt[3]{e} =$

d) $\log_{\frac{2}{3}} \frac{27}{8} =$

e) $\log_{10} 10 =$

f) $\ln 1 =$

2. Use properties of logs and write the following logs in expanded form.

a) $\log_2 \left(\frac{2ab}{c^3} \right) =$

b) $\ln \left[(x-4)(2x+5) \right]^2 =$

c) $\ln \left[(x-4)(2x+5)^2 \right] =$

d) $\log_8 \sqrt{xy} =$

3. Express each of the following as a single log.

a) $3\log_5 x + \log_5 y - 2\log_5 W =$

b) $\frac{1}{2} \left[(2\ln a + \ln b) - 5 \ln c \right] =$

c) $\frac{1}{2} \ln x - \frac{1}{3} \ln y =$

4. Solve the following equations:

a) $\log_2(x+1) + \log_2 3 = 1$

b) $\log_2(x+3) + \log_2 x = 2$

c) $\ln(x-4) + \ln x = \ln 21$

d) $4\ln(3x) = 4$

e) $\ln x = \ln(1-x)$

f) $7 + 2\ln x = 6$

g) $\log x + \log(x-1) = \log(3x+12)$

h) $\log_2(x^2-6x) = 3 + \log_2(1-x)$

i) $5 + e^{x+1} = 20$

j) $2^x = 7$

k) $4^{x-3} = 9$

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5) Simplify each log.

a) $\ln e^5 =$

b) $e^{2\ln 5} =$

c) $10^{2+\log 5} =$

d) $\dfrac{\log 100}{\log 10} =$

e) $\dfrac{\log_3 9}{\log_2 8} =$

Solutions

(1)

a) $\log_2 16 = 4 \longleftrightarrow 2^4 = 16$

b) $\log_7 1 = 0 \longleftrightarrow 7^0 = 1$

c) $\ln \sqrt[3]{e} = \ln e^{\frac{1}{3}} = \frac{1}{3} \ln e = \frac{1}{3}$

d) $\log_{\frac{2}{3}} \frac{27}{8} = -3 \longleftrightarrow \left(\frac{2}{3}\right)^{-3} = \frac{27}{8}$

$2^3 = 8$, $3^3 = 27$ $\Rightarrow \left(\frac{2}{3}\right)^3 = \frac{8}{27} \rightsquigarrow \left(\frac{2}{3}\right)^{-3} = \frac{27}{8}$

flip the fraction means change the power sign.

e) $\log_{10} 10 = 1 \longleftrightarrow 10^1 = 10$

f) $\ln 1 = 0 \longleftrightarrow e^0 = 1$

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2)

a) $\log_2 \frac{2ab}{c^3} = \log_2 2ab - \log_2 c^3$

$= \log_2 2 + \log_2 a + \log_2 b - 3\log_2 c = 1 + \log_2 a + \log_2 b - 3\log_2 c$

b) $\ln\left[(x-4)(2x+5)\right]^2 = 2\ln\left[(x-4)(2x+5)\right]$

$= 2\left[\ln(x-4) + \ln(2x+5)\right]$

c) $\ln\left[(x-4)(2x+5)^2\right] = \ln(x-4) + \ln(2x+5)^2$

$= \ln(x-4) + 2\ln(2x+5)$

d) $\log_8 \sqrt{xy} = \log_8 (xy)^{\frac{1}{2}} = \frac{1}{2}\log_8 xy = \frac{1}{2}\left[\log_8 x + \log_8 y\right]$

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3)

a) $3\log_5 x + \log_5 y - 2\log_5 w = \log_5 x^3 + \log_5 y - \log_5 w^2$

$= \log_5 \frac{x^3 \cdot y}{w^2}$

b) $\frac{1}{2}\left[2\ln a + \ln b - 5\ln c\right] = \frac{1}{2}\left[\ln a^2 + \ln b - \ln c^5\right]$

$= \frac{1}{2}\left[\ln \frac{a^2 \cdot b}{c^5}\right]$

$= \ln \sqrt{\frac{a^2 b}{c^5}} = \ln \frac{\sqrt{a^2}\cdot\sqrt{b}}{\sqrt{c^5}} = \ln \frac{a\sqrt{b}}{c^2\sqrt{c}}$

$c^4 \cdot c$

c) $\frac{1}{2}\ln x - \frac{1}{3}\ln y = \ln x^{\frac{1}{2}} - \ln y^{\frac{1}{3}}$

$= \ln\sqrt{x} - \ln\sqrt[3]{y} = \ln\frac{\sqrt{x}}{\sqrt[3]{y}}$

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4) a) $\log_2(x+1) + \log_2 3 = 1$

$\log_2 3(x+1) = 1 = \log_2 2 \xrightarrow[\text{of LOG form}]{\text{make both sides}} 3(x+1) = 2$

same base

$\Rightarrow 3x + 3 = 2$

$\Rightarrow 3x = 2 - 3$ $\quad -1$

$\Rightarrow \boxed{x = \frac{-1}{3}}$

b) $\log_2^{x+3} + \log_2^{x} = 2$

$\log_2 (x+3)x = 2 = \log_2 4 \Rightarrow x(x+3) = 4 \Rightarrow x^2 + 3x - 4 = 0$

same base

$\Rightarrow (x+4)(x-1) = 0$

$\Rightarrow \boxed{x = -4 \;,\; x = 1}$

c) $\ln(x-4) + \ln x = \ln 21$

$\ln x(x-4) = \ln 21 \Rightarrow x^2 - 4x - 21 = 0$

$\Rightarrow (x-7)(x+3) = 0$

$\Rightarrow \boxed{x = 7 \;,\; x = -3}$

d) $4\ln(3x) = 4 \Rightarrow \ln(3x) = 1 = \ln e \Rightarrow 3x = e \Rightarrow \boxed{x = \frac{e}{3}}$

e) $\ln x = \ln(1-x)$

$\xrightarrow[\text{from both sides}]{\text{take } e} \quad e^{\ln x} = e^{\ln(1-x)}$

$\xrightarrow[e \,\&\, \ln]{\text{cancel}} \quad x = 1-x \Rightarrow 2x = 1 \Rightarrow \boxed{x = \frac{1}{2}}$

f) $7 + 2\ln x = 6 \Rightarrow 2\ln x = 6-7 = -1$

$\Rightarrow \ln x = \frac{-1}{2}$

$\xrightarrow[\text{from both sides}]{\text{take } e} \quad e^{\ln x} = e^{-\frac{1}{2}} \Rightarrow x = e^{-\frac{1}{2}} = \frac{1}{e^{\frac{1}{2}}} = \boxed{\frac{1}{\sqrt{e}}}$

g) $\ln x + \ln(x-1) = \ln(3x+12)$

$\Rightarrow \ln x \cdot (x-1) = \ln 3x+12$

$\xrightarrow[\text{from both sides}]{\text{take } e} \quad e^{\ln x(x-1)} = e^{\ln 3x+12}$

$\xrightarrow[e \,\&\, \ln]{\text{cancel}} \quad x^2 - x = 3x + 12 \Rightarrow x^2 - 4x - 12 = 0$

$\Rightarrow (x-6)(x+2) = 0$

$\Rightarrow \boxed{x = 6 \;,\; x = -2}$

h) $\log_2 x^2 - 6x = 3 + \log_2 1-x$

$\log_2 x^2 - 6x - \log_2 1-x = 3 = \log_2 8$

$\log_2 \frac{x^2-6x}{1-x} = \log_2 8 \Rightarrow \frac{x^2-6x}{1-x} = \frac{8}{1}$

$\Rightarrow x^2 - 6x = 8 - 8x$

$\Rightarrow x^2 + 2x - 8 = 0$

$\Rightarrow (x+4)(x-2) = 0 \Rightarrow \boxed{x = -4 \,,\, 2}$

i) $5 + e^{x+1} = 20 \Rightarrow e^{x+1} = 20 - 5 = 15$

$\xrightarrow[\ln \text{ from both sides}]{\text{take}} \quad \ln(e^{x+1}) = \ln 15$

$\xrightarrow[e \,\&\, \ln]{\text{cancel}} \quad x+1 = \underbrace{\ln 15}_{\text{a number}} \Rightarrow \boxed{x = (\ln 15) - 1}$

j) $2^x = 7 \xrightarrow[\text{from the both sides}]{\text{base is } = 2 \text{, so take } \log_2} \log_2 2^x = \log_2 7 \Rightarrow x \log_2 2 = \log_2 7 \Rightarrow \boxed{x = \underbrace{\log_2 7}_{\text{a number}}}$

K) $4^{x-3} = 9 \xrightarrow[\text{of } 4^x]{\text{take } \log_4 \text{ to get rid}} \log_4 4^{x-3} = \log_4 9$

$\Rightarrow (x-3)\log_4 4 = \log_4 9$

$\Rightarrow x - 3 = \log_4 9 \Rightarrow \boxed{x = \log_4 9 + 3}$

5) a) $\cancel{\ln} \cancel{e}^5 = 5$

b) $e^{2\ln 5} = \cancel{e}^{\cancel{\ln} 5^2} = 5^2 = 25$

ignore (c) & (d)

e) $\dfrac{\log_3 9}{\log_2 8} = \dfrac{\log_3 3^2}{\log_2 2^3} = \dfrac{2\cancel{\log_3 3}}{3\cancel{\log_2 2}} = \dfrac{2}{3}$