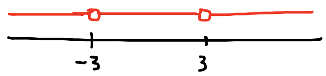
1)

a) $f(x) = \dfrac{x+2}{x^2-9}$

Find 0's of the denominator : $x^2 - 9 = 0 \Rightarrow x^2 = 9 \Rightarrow x = 3, -3 \Rightarrow$ EXCLUDE



$$\text{Dom} = (-\infty,-3) \cup (-3,3) \cup (3,+\infty)$$ $\longrightarrow$ Interval notation

b) $g(x) = x^7 + x^2 - 1$

$\text{Dom} = \mathbb{R} = (-\infty, +\infty)$ interval

c) $h(t) = \dfrac{\sqrt{2t-1}}{t^3+1}$

numerator : $\sqrt{\bigcirc} \longrightarrow$ must be non-negative ($\geqslant 0$)

denominator : $\neq 0 \longrightarrow t^3+1 = 0 \Rightarrow t^3 = -1 \Rightarrow t = -1$ EXCLUDE

$\longrightarrow 2t - 1 \geqslant 0 \Rightarrow 2t \geqslant 1 \Rightarrow t \geqslant \frac{1}{2}$

We can ignore -1 because it is not greater than $\frac{1}{2}$.

$$\Rightarrow \text{Dom} = \left[\frac{1}{2}, \infty\right)$$

d) $f(t) = \dfrac{t^4+1}{\sqrt{t(t-1)}}$ $\longrightarrow$ NO restriction (numerator)

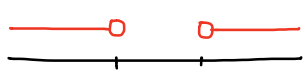
$\longrightarrow$ Can NOT be 0 and negative (denominator)

$\Rightarrow$ must be strictly positive.

$\Rightarrow t(t-1) > 0$

		0		1	
t	$-$	×	$+$		$+$
$t-1$	$-$		$-$	×	$+$
$t(t-1)$	$+$		$-$		$+$
	$\downarrow$				$\downarrow$
	$t<0$				$t>1$

$$\text{Dom} = (-\infty, 0) \cup (1, \infty)$$



2) $u(x) = \sqrt{2x+3}$, $f(x) = |x^2 - 1|$

a)

- $f \circ u(0) = f(u(0)) = f(\sqrt{3}) = |(\sqrt{3})^2 - 1| = |3-1| = |2| = 2$

- $f \circ u(-1) = f(u(-1)) = f(\sqrt{2\times(-1)+3}) = f(\sqrt{\cancel{-2+3}}) = f(1) = |1^2 - 1| = |0| = 0$

  $\sqrt{1} = 1$

- $u \circ f(-\frac{1}{2}) = u(f(-\frac{1}{2})) = u(\frac{3}{4}) = \sqrt{2\times\frac{3}{4}+3} = \sqrt{\frac{6}{4}+3} = \sqrt{\frac{6+12}{4}} = \sqrt{\frac{18}{4}} = \frac{3\sqrt{2}}{2}$

  just simplification

  $* \, f(-\frac{1}{2}) = |(-\frac{1}{2})^2 - 1| = |\frac{1}{4} - 1| = |-\frac{3}{4}| = \frac{3}{4}$

- $u \circ f(0) = u(f(0)) = u(1) = \sqrt{2\times1+3} = \sqrt{5}$

  $* \, f(0) = |0^2 - 1| = |-1| = 1$

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b) $f \circ u(x) = f(u(x)) = |(u(x))^2 - 1| = |(\cancel{\sqrt{2x+3}})^{\cancel{2}} - 1|$

$= |2x+3-1|$

$= |2x+2|$

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c) $f \circ u(x) = |2(x+1)| = 2|x+1|$

The absolute value function must be split into two parts where $\boxed{x+1}$ is positive and negative. You see that $x+1$ is equal to 0 when $x=-1$ and for any number greater than -1, $x+1$ is positive and for numbers less than -1, $x+1$ is negative. Thus

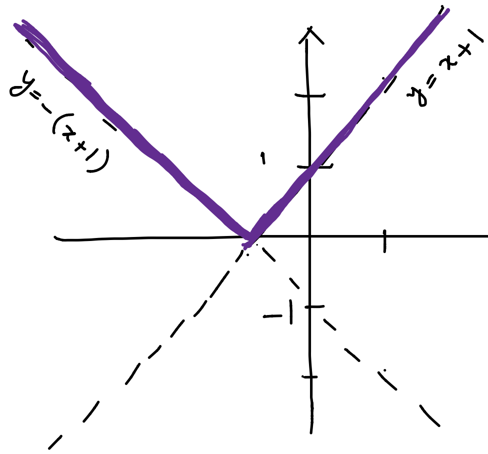
$$|x+1| = \begin{cases} x+1 & x \geqslant -1 \quad (x+1 \geqslant 0) \\ -(x+1) & x < -1 \quad (x+1 < 0) \end{cases}$$

graph: $y = x+1$

x	y
0	1
1	1+1=2

$y = -(x+1)$

x	y
0	$-(0+1)=-1$
1	$-(1+1)=-2$



Domain $= (-\infty, \infty)$

Range $= [0, \infty)$

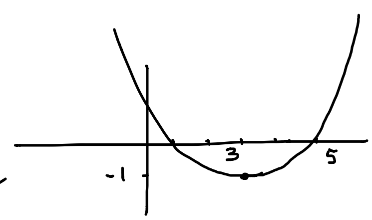
a) crossing x-axis at 5 $\rightsquigarrow$ x-intercept $= 5$ $\to$ $(5,0)$ is on f

$\xrightarrow{f^{-1}}$ y-intercept $= 5$ $\rightsquigarrow$ $(0,5)$ on f^{-1}.

b) f quadratic $\rightsquigarrow$ graph: parabola

vertex: $(3,-1)$

from (a) x-int $= 5$



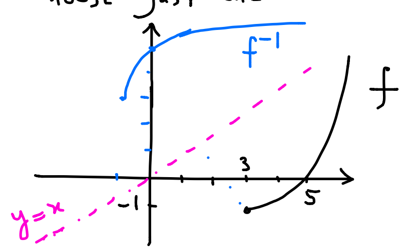
NOT 1-1 thus NOT invertible

To make it invertible:

Cut the graph from the vertex and choose just one side (either left or right)

For f: Dom: $[3, \infty)$

Range: $[-1, \infty)$



For f^{-1}: (switch dom & Range)

$\Rightarrow$ Dom: $[-1, \infty)$

Range: $[3, +\infty)$

reflect the graph of f over the line $y=x$

4) P and Q are on both graphs, they satisfy both equations
so:

$(x-1)^2+1 = -2x^2+3$

$(x^2-2x+1)+1 = -2x^2+3$

$\Rightarrow \underline{x^2}-2x+2+\underline{2x^2}-3 = 0$

$\Rightarrow \underline{3}x^2-2x\,\underline{-1} = 0$

$ac=-3$
$b=-2$ $\Rightarrow$ look for two numbers: their product $= -3$ $\Rightarrow$ -3 and 1
sum $= -2$

break the x-term into two terms with -3 and 1:

$3x^2-3x+x-1 = 0$

$3x(x-1)+(x-1) = 0$

$(x-1)(3x+1) = 0 \Rightarrow$ $x-1=0 \Rightarrow x=1$
$3x+1=0 \Rightarrow x=\frac{-1}{3}$

To find y-coordinates, plug the two x's into the equations (choose any):

$y=(x-1)^2+1$
$\nearrow x=1 \Rightarrow y=(1-1)^2+1=1$
$\searrow x=\frac{-1}{3} \Rightarrow y=\left(-\frac{1}{3}-1\right)^2+1=\left(-\frac{4}{3}\right)^2+1=\frac{16}{9}+1=\frac{25}{9}$

$$\Rightarrow \boxed{P=(1,1), \quad Q=\left(-\frac{1}{3}, \frac{25}{9}\right)}$$

b) slope $m=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{1-\frac{25}{9}}{1-\left(-\frac{1}{3}\right)}=\dfrac{\frac{9-25}{9}}{\frac{3+1}{3}}=\dfrac{\frac{-16}{9}}{\frac{4}{3}}=-\dfrac{\cancel{3}^{1}\times\cancel{16}^{4}}{{}_{3}\cancel{9}\times\cancel{4}_{1}}=-\dfrac{4}{3}$

$y-y_2=m(x-x_2) \Rightarrow y-1=-\frac{4}{3}(x-1)$

$\Rightarrow y=-\frac{4}{3}x+\frac{4}{3}+1 \Rightarrow \boxed{y=-\frac{4}{3}x+\frac{7}{3}}$

$y=(x-1)^2+1=x^2-2x+2$

vertex: $x=\frac{-b}{2a}=\frac{-(-2)}{2\times1}=1$
$y=(1-1)^2+1=1$ $\Rightarrow (1,1)$

y-int: $y=2 \rightsquigarrow (0,2)$

$y=-2x^2+3 \rightarrow b=0$

vertex $x=\frac{-b}{2a}=\frac{0}{2\times-2}=0$
$y=-2\times0+3=3$ $\Rightarrow (0,3)$
also
y-int

$x=1 \Rightarrow y=-2\times1^2+3$
$=1$ $\Rightarrow (1,1)$

$y = -\frac{4}{3}x + \frac{7}{3}$

We already have two points P and Q.