①

Solutions to the selected problems

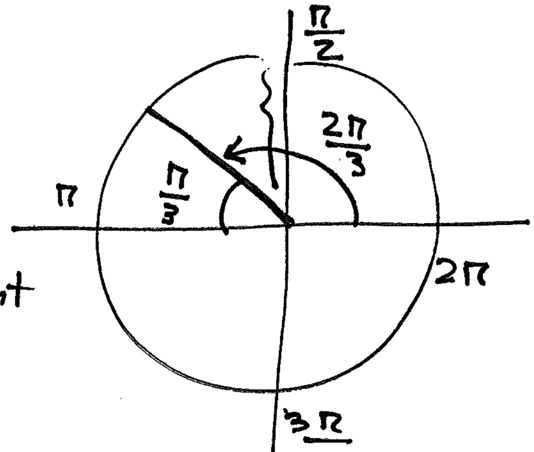
(e) $$\lim_{\theta \to \frac{\pi}{4}} \theta \tan\theta = \frac{\pi}{4} \underbrace{\cancel{\tan}\frac{\pi}{4}}_{1}$$

$$= \frac{\pi}{4}$$

(f) $$\lim_{x \to 2\pi} \left(\sin \frac{x}{3}\right)^2 = \lim_{x \to 2\pi} \left(\sin \frac{2\pi}{3}\right)^2$$

$$\frac{2\pi}{3} = \frac{3\pi - \pi}{3}$$

$$= \pi - \frac{\pi}{3} \to 2^{nd} \text{ quadrant}$$

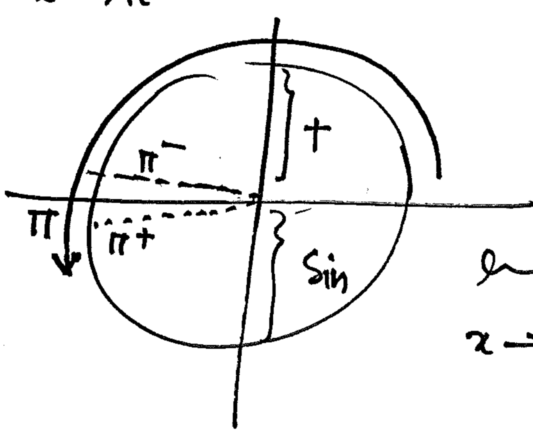


$$\sin \frac{2\pi}{3} = + \frac{\sqrt{3}}{2}$$

$$= \lim_{x \to 2\pi} \left(\sin \frac{2\pi}{3}\right)^2 = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$$

(h) $$\lim_{x \to \pi} \frac{\sin x}{\sec x} = \frac{\sin \pi}{\frac{1}{\cos \pi}} = \frac{0}{-1} = 0$$

(i) $$\lim_{x \to \pi^+} \frac{\sec x}{\sin x} = \frac{\frac{1}{-1\ \cancel{\cos}\pi}}{\underbrace{\sin \pi}_{-}} = \frac{-1}{0^-} = +\infty$$



$$\lim_{x \to \pi^-} \frac{\sec x}{\sin x} = \frac{-1}{0^+} = -\infty$$

l) $\lim_{x\to 6^-} \ln(6-x) = \ln(6 - \underset{5.9}{6}) = \ln 0^+$

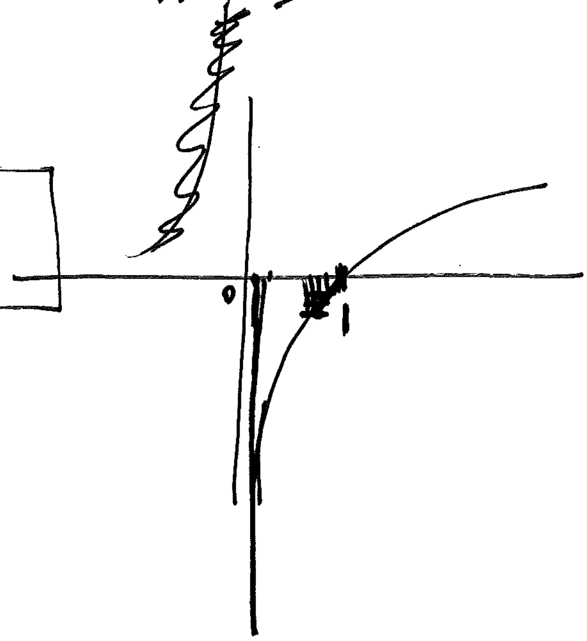
= NOT defined

$= -\infty$

ln ▨ is defined just when ▨ > 0

Domain: $6 - x > 0$

$$\boxed{x < 6}$$



m) $\lim_{x\to 1^-} e^x + \frac{1}{\ln x}$

$$= e^1 + \frac{1}{\ln 1} = \cancel{e} + \frac{1}{0^-} = -\infty$$

$\cancel{e - \infty} = -\infty$

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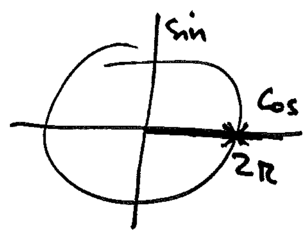
(2) (f) Check $\lim_{x\to\pi^-} g(x) \overset{?}{=} \lim_{x\to\pi^+} g(x) \overset{?}{=} g(\pi)$

$$\lim_{x\to\pi^-} g(x) = \lim_{x\to\pi^-} e^{2\sin 2x} = e^{2\overset{0}{\sin 2\pi}} = e^0 = 1$$

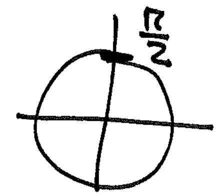
$= g(\pi)$



$$\lim_{x \to \pi^+} g(x) = \lim_{x \to \pi^+} \ln\left(\sin\left(x - \frac{\pi}{2}\right)\right)$$

$$= \ln\left(\sin\left(\pi - \frac{\pi}{2}\right)\right)$$

$$= \ln\left(\underbrace{\sin \frac{\pi}{2}}_{1}\right) = \ln 1 = 0$$



$$\Rightarrow \lim_{x \to \pi^+} g(x) \neq \lim_{x \to \pi^-} g(x) \Rightarrow \text{NOT cont. at } x = \pi$$

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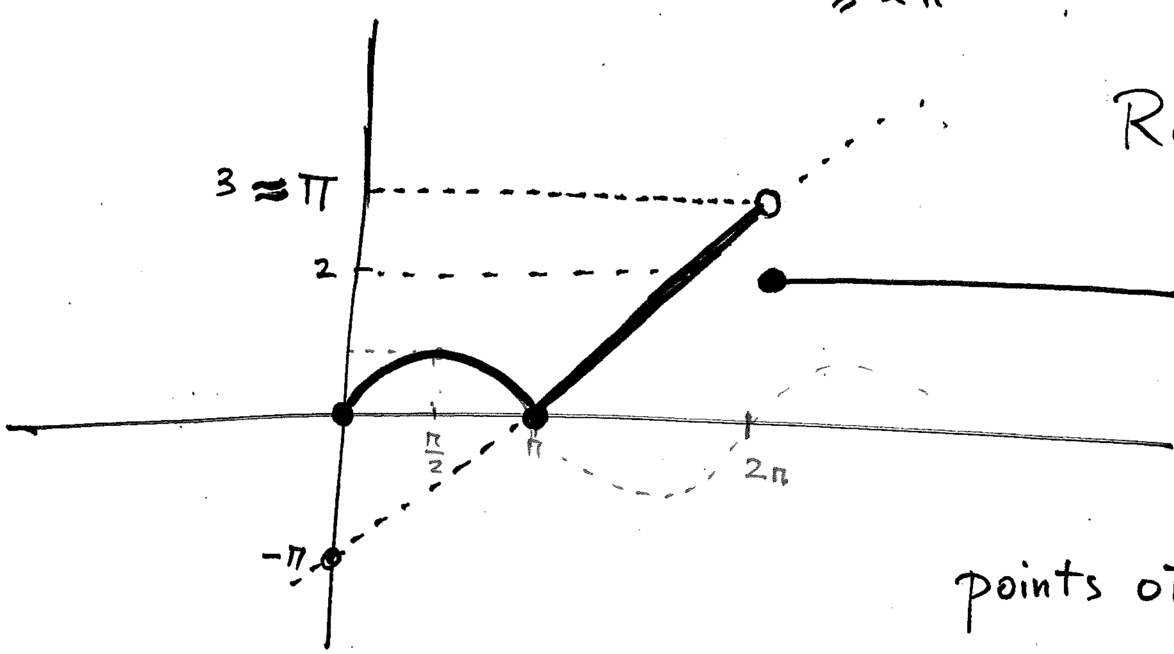
(3) (a) $\begin{cases} \sin x & 0 \le x < \pi \\ x - \pi & \pi \le x < 2\pi \\ 2 & x \geq 2\pi \end{cases}$

Domain: $[0, +\infty)$



Range: $[0, \pi)$

$y = x - \pi$

$2\pi - \pi = \pi$

x	y
0	$-\pi$
π	0

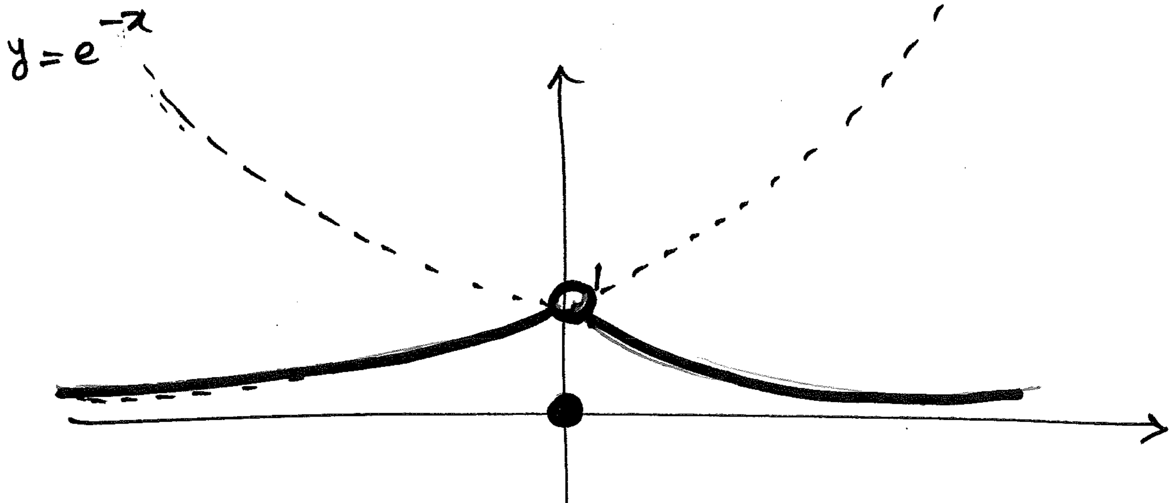
points of non-diffability:

$x = \pi, \ 2\pi$

$x = 0$ because it has only the right part.

$$g(x) = \begin{cases} e^{x} & x < 0 \\ 0 & x = 0 \\ e^{-x} & x > 0 \end{cases}$$

$y = e^{x}$



x-int : None

y-int : 0 (NOT 1)

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④ IVT

(a) $y = x^4 + 2x^2 - 4x - 1$

$$\boxed{y' = 4x^3 + 4x - 4 = 0}$$

y' is cont's everywhere

IVT for y'

Show at least one point that $y' = 0$

$\rightsquigarrow$ at least one root for y' $\longrightarrow$ $\boxed{\text{IVT}}$

Apply IVT to y' :

x	0	Root	1
y'	-4		$4+4-4=+4$

By IVT there is a (c) root in $[0,1]$

(d) some u $0 < u < 2$

such that $u^2 + \cos(\pi u) = 4$

$f(x) = x^2 + \cos(\pi x) - 4$

If I show there is a point u such that

$f(u) = 0 \longrightarrow$ I solved the questions

$\downarrow$

$u^2 + \underline{\cos(\pi u)} - 4 = 0$

cont is everywhere

$f(x)$ is cont is everywhere

x	0	1	2
$f(x)$	$0 + \underbrace{\cos 0}_{1} - 4$	$1 + \overbrace{\cos \pi}^{-1} - 4$	$\cancel{4} + \cos(2\pi) - \cancel{4}$
	-3	-4	$+1$

sign change: by IVT

there is a number u in $[0, 2]$

such that $f(u) = 0$

$\Rightarrow u^2 + \cos(\pi u) - 4 = 0$

$\Rightarrow u^2 + \cos(\pi u) = 4$

Remark: Sin & Cos are always a number between (−1) and 1 (inclusive) so:

$\sin x = -2 \rightsquigarrow$ NO solution

$\cos x = 3 \rightsquigarrow$ NO "

$\sin 2\theta = 5 \rightsquigarrow$ NO "

$\cos 3\theta = 3/2 \rightsquigarrow$ NO "

⑥ (a) $f(x) = x^2 \sin 2x$

x-int : $y = 0$

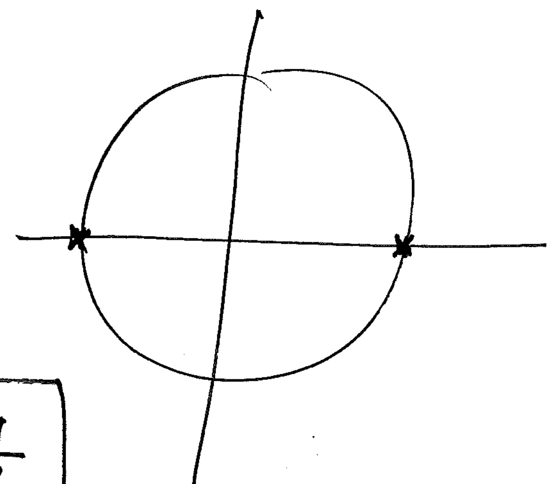
$$x^2 \sin 2x = 0$$

$$x^2 = 0 \longrightarrow \boxed{x = 0}$$

or

$$\sin(2x) = 0$$

$2x \to \theta$



$$\theta = \pi, \ 2\pi$$

$$2x = \theta = \pi \Rightarrow \boxed{x = \frac{\pi}{2}}$$

$$2x = \theta = 2\pi \Rightarrow \boxed{x = \pi}$$

in $[0, 2\pi]$ (one cycle)

(C) $(\cos x) \ln x = 0$ in $[0, 2\pi]$

$$\cos x = 0$$

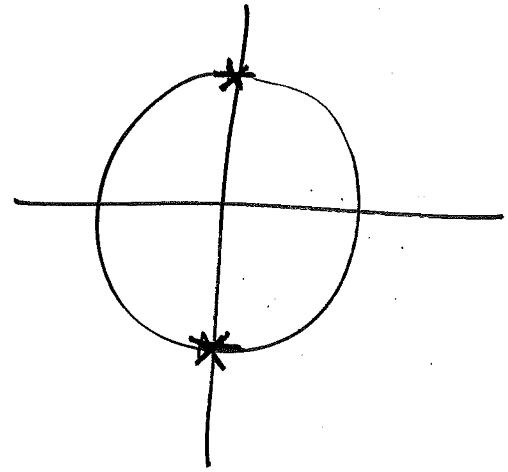
$$\ln x = 0 \longrightarrow \text{~~NO Solution~~} \ \boxed{x = 1}$$

$\cancel{\ln | 0}$

$\cos x = 0$

$\boxed{x = \frac{\pi}{2}}$

or $\boxed{x = \frac{3\pi}{2}}$



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(b) $2e^x + 5 = 115$

$2e^x = 115 - 5 = 110$

$\cancel{\ln}(\cancel{e}^x) = \frac{110}{2} = \ln(55)$

$\boxed{x = \ln 55}$

(d) $e^{\ln(x+1)} = e^2$

$\cancel{e}^{\cancel{\ln}(x+1)} = e^2$

$x+1 = e^2 \Rightarrow \boxed{x = e^2 - 1}$

(8)

(1) $$\boxed{\lim_{h \to 0} \frac{\tan(3x+3h) - \tan 3x}{h}} = f'(x)$$

Compare

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x) = \tan 3x$$

$$f'(x) = \sec^2 3x \cdot 3$$

(3) $$\lim_{h \to 0} \frac{8\left(\frac{1}{2}+h\right)^8 - 8\left(\frac{1}{2}\right)^8}{h} = f'(a)$$

Compare

$$f'(a) = \lim_{h \to 0} \frac{f(\overset{\frac{1}{2}}{a}+h) - f(\overset{\frac{1}{2}}{a})}{h}$$

$$f(x) = 8x^8 \xrightarrow{f'(\frac{1}{2})} f'(x) = 64x^7$$

$$a = \frac{1}{2}$$

$$f'\left(\frac{1}{2}\right) = 64 \cdot \left(\frac{1}{2}\right)^7 = \frac{2^{\cancel{6}}}{2^{\cancel{7}}} = \frac{1}{2}$$

(14) $y = e^{ax} + b(x+1)^3$

$x = 0 \rightsquigarrow \begin{matrix} y' = 0 \\ y'' = 0 \end{matrix} \longrightarrow a, b \;?$

$y'(0) = 0$

$y''(0) = 0$

$y'(x) = e^{ax} \cdot a + b\,3(x+1)^2 \cdot 1$

$= ae^{ax} + 3b\,(x+1)^2$

$y'(0) = ae^0 + 3b\,(0+1)^2 = 0$

$= \boxed{a + 3b = 0}$

$y''(x) = a\,e^{ax} \cdot a + 3b \cdot 2(x+1)^1 \cdot 1$

$= a^2 e^{ax} + 6b\,(x+1)$

$y''(0) = \boxed{a^2 + 6b = 0}$

$$\begin{cases} a + 3b = 0 \Rightarrow a = -3b \\ a^2 + 6b = 0 \end{cases}$$

$a=-3b$

$$(-3b)^2 + 6b = 0$$

$$\Rightarrow 9b^2 + 6b = 0$$

$$b(9b+6) = 0 \Rightarrow \boxed{b=0}$$

$$9b+6=0$$

$$\Rightarrow \boxed{b = -\frac{6}{9} = -\frac{2}{3}}$$

$$a = -3b \quad \nearrow \; a = -3 \cdot 0 = 0$$

$$\searrow \; a = -3 \cdot -\frac{2}{3} = +2$$

Either $\boxed{a=0, \; b=0}$

or $\boxed{a=2, \; b=-\frac{2}{3}}$

(16) (a) $$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

(b) $f'(2)$ when

$$f(x) = x + \frac{1}{x}$$

$$f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \to 0} \frac{2+h + \frac{1}{2+h} - \left(2 + \frac{1}{2}\right)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2}+h + \frac{\boxed{1}}{2+h} - \cancel{2} - \frac{\boxed{1}}{2}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{2h(2+h) + \cancel{2} - \overset{-\cancel{2}-h}{(2+h)}}{2(2+h)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{2h(2+h) - h}{2(2+h)}}{\frac{h}{1}}$$

top

bottom

$$= \lim_{h \to 0} \frac{4h + 2h^2 - h}{2h(2+h)}$$

$$= \lim_{h \to 0} \frac{3h + 2h^2}{2h(2+h)} = \frac{0}{0}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(3+2h)}{2\cancel{h}(2+h)} = \frac{3+0}{2(2+0)}$$

$$\boxed{= \frac{3}{4}}$$

From the derivative rules check your work :

$$f(x) = x + \frac{1}{x} = x + x^{-1}$$

$$f'(2) = \frac{3}{4} \;?$$

$$f'(x) = 1 + (-1)\, x^{\cancel{-1-1}\;-2} = 1 - \frac{1}{x^2}$$

$$f'(2) = 1 - \frac{1}{2^2} = 1 - \frac{1}{4} = \frac{3}{4} \checkmark$$

$=$

(26) Exp Growth/Decay

(b) $y(t) = y_0 e^{-kt}$

decreases proportional to ... Exp model

$y_0 = 6$ grams

$t = 1 \rightsquigarrow y(1) = 1$ gram

Half-life ? $T_{\frac{1}{2}} = \frac{\ln 2}{k}$

Use the info

$1 = 6 e^{-k} \Rightarrow \frac{1}{6} = e^{-k}$

$\Rightarrow \ln \frac{1}{6} = -k$

$\Rightarrow \boxed{k = -\ln \frac{1}{6}} = +$

$\ominus$

positive or negative ?

$\boxed{\text{K ALWAYS Positive}}$

$$\boxed{T_{\frac{1}{2}} = \frac{\ln 2}{-\ln \frac{1}{6}}}$$

$$y(t) = y_0 e^{-kt}$$

$$y(t) = 6 e^{+\ln \frac{1}{6} t}$$

complete the model

decay rate $\longrightarrow$ $y'(0) = ?$

initially

$t = 0$

$$y'(t) = 6 \underbrace{e^{\ln \frac{1}{6} t}}_{\text{outside}} \cdot \underbrace{\ln \frac{1}{6}}_{\text{inside}}$$

$$y'(0) = 6 \cancel{e^{\ln \frac{1}{6} \cdot 0}} \cdot \ln \frac{1}{6}$$

$$\boxed{= 6 \ln \frac{1}{6}}$$

(e) Doubling time $T_2 = \frac{\ln 2}{k}$?

$$y(t) = y_0 e^{kt}$$

$$y_0 \xrightarrow{9 \text{ hrs}} 3y_0$$

$$3\cancel{y_0} = \cancel{y_0} e^{9k}$$

$$3 = e^{9k} \longrightarrow \ln 3 = 9k \rightarrow k = \frac{\ln 3}{9}$$

$$\boxed{T_2 = \frac{\ln 2}{\frac{\ln 3}{9}} = \frac{9 \ln 2}{\ln 3}}$$

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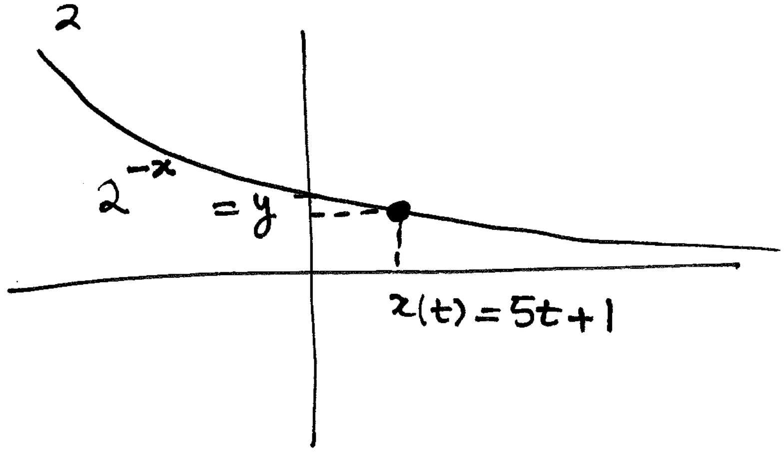


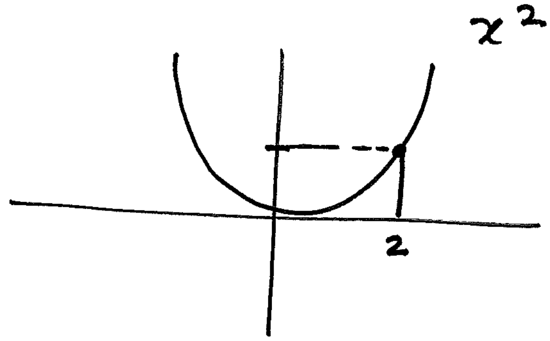
$$x(t) = 5t+1$$

$$y(t) = 2^{-x(t)}$$

$$= 2^{-(5t+1)}$$

$$= 2$$

$$y'(t) = \underbrace{2^{-(5t+1)} \cdot \ln 2}_{\text{outside}} \cdot \underbrace{-5}_{\text{inside}}$$



$$y'(t) = -5 \ln 2 \cdot 2^{-(5t+1)}$$