Worksheet Oct 27

(1) Let $f(x)$ be the function given by

$$f(x) = \underbrace{5\sqrt[3]{x}}_{h}\,\underbrace{e^{x}}_{g}$$

Find the slope of the tangent line at each of the following x's :

$m_{tan} = f'(x)$ at the given points

$x = 1$, $x = -1$

You have product of two functions : there are different ways to break $f(x)$ into two function.

$$5\sqrt[3]{x} \qquad e^{x}$$

$$\sqrt[3]{x} \qquad 5e^{x}$$

Or you can take 5 out , because it's a constant and do the differentiation and multiply it by the final result.

$$f(x) = 5x^{\frac{1}{3}}\, e^{x}$$

$$f'(x) = \overbrace{5\left(\frac{1}{3}x^{\frac{1}{3}-1}\right)}^{h'}\overbrace{e^{x}}^{g} + \overbrace{\left(5x^{\frac{1}{3}}\right)}^{h}\overbrace{e^{x}}^{g'}$$

$$= \frac{5}{3}x^{-\frac{2}{3}}e^{x} + 5x^{\frac{1}{3}}e^{x}$$

$$= \frac{5}{3\sqrt[3]{x^{2}}}e^{x} + 5\sqrt[3]{x}\,e^{x}$$

make the fraction powers and negative powers into roots with positive powers when you want to plug a number into x.

(2) Find the equation of the tangent line to the curve $y = \dfrac{x-1}{\sqrt{x}}$ at $x = 4$.

point : $x = 4 \rightsquigarrow y = \dfrac{4-1}{\sqrt{4}} = \dfrac{3}{2} \rightsquigarrow \left(4, \dfrac{3}{2}\right)$

slope : $m_{tan} = y'(4)$

Use quotient rule to find $y'(x)$:

$$y'(x) = \frac{\overbrace{1}^{f'}\,\overbrace{(\sqrt{x})}^{g} - \overbrace{\frac{1}{2}x^{\frac{1}{2}-1}}^{g'}\,\overbrace{(x-1)}^{f}}{\underbrace{(\sqrt{x})^2}_{g^2}}$$

$$= \frac{\sqrt{x} - \frac{1}{2\sqrt{x}}(x-1)}{x}$$

No need to simplify more because we'll plugin numbers.

$$y'(4) = \frac{\sqrt{4} - \frac{1}{2\sqrt{4}}(4-1)}{4} =$$

$$= \frac{2 - \frac{1}{4}3}{4}$$

$$= \frac{2 - \frac{3}{4}}{4} = \frac{\frac{5}{4}}{4} = \frac{5}{16}$$

$$\frac{\frac{5}{4}}{\frac{4}{1}} = \frac{5}{16}$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \times d}{b \times c}$$

$m_{\tan}$ at $x=1$ $= f'(1)$

$$= \frac{5}{3\,\underbrace{\cancel{\sqrt[3]{1^2}}}_{1}}\, e^{1} + 5\,\underbrace{\cancel{\sqrt[3]{1}}}_{1}\, e^{1}$$

$$= \frac{5}{3} e + 5e$$

$$= \boxed{\frac{20}{3} e}$$

$m_{\tan}$ at $x=-1$ $= f'(-1)$

$$= \frac{5}{3\left(\underbrace{\cancel{\sqrt[3]{(-1)^2}}}_{1}\right)}\, \overbrace{e^{-1}}^{\frac{1}{e}} + 5\,\underbrace{\cancel{\sqrt[3]{(-1)}}}_{-1}\, e^{-1}$$

$$= \frac{5}{3}\,\frac{1}{e} + 5(-1)\,\frac{1}{e}$$

$$= \boxed{\frac{-10}{3e}}$$