HW # 2 : Solution:

1) $(x+1)^2 + (y-1)^2 = 9$

a) slope of the tangent line $= y'$

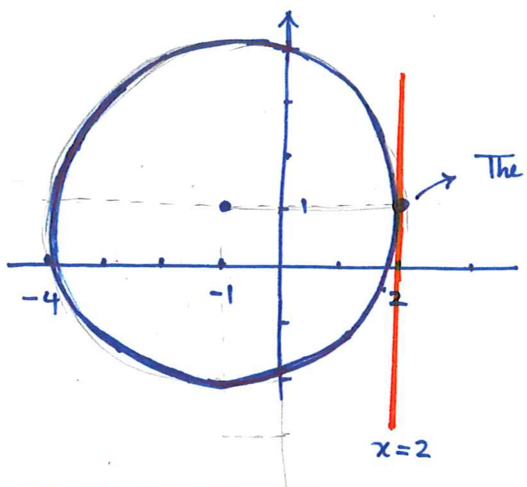
$\xrightarrow{\text{Differentiate}}$ $2(x+1) + 2(y-1)y' = 0 \Rightarrow y' = \dfrac{-2(x+1)}{2(y-1)}$

$\Rightarrow y' = \dfrac{-(x+1)}{y-1}$

When $y = +1 \Rightarrow y' = \dfrac{-(x+1)}{\underbrace{1-1}_{0}} =$ undefined $= \infty$

$\Rightarrow$ Slope is NOT defined $\Rightarrow$ Vertical tangent line.

b)



The point with $y=1$ and positive x-coordinate

plug $y=1$ into circle

$(x+1)^2 = 9$

$\Rightarrow$ $x+1 = 3 \Rightarrow x = 2 \rightarrow$ positive

$x+1 = -3 \Rightarrow x = -4$

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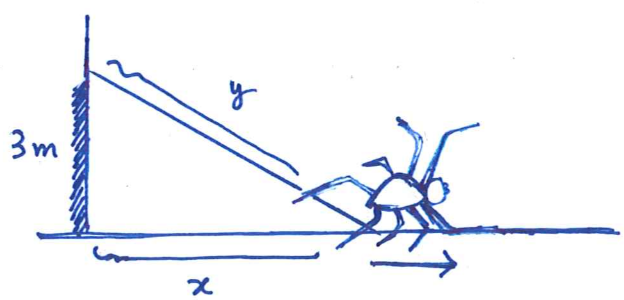
2)



$\dfrac{dx}{dt} = x'(t) = 10$ cm/sec

$t = 40$ Sec

* Unify all the measurement units first : 3 m $= 300$ cm

Relate Variables: $x^2 + (300)^2 = y^2$

Take the derivative of both sides with respect to t:

$$\left((x(t))^2 + (300)^2 = (y(t))^2 \right)'$$

$$2\underline{x}\,\underbrace{x'}_{10} + 0 = 2\underline{y}\,y' \text{ unknown} \qquad (*)$$

Compute first x and y:

- x is changing 10 cm per second so after $t = 40$ sec

$$\Rightarrow \quad x = 10 \times 40 = 400 \text{ Cm}$$

- Use Pythagorean Theorem to find y

$$\Rightarrow y^2 = (400)^2 + (300)^2 = 250000$$

$$\Rightarrow y = 500$$

sub into $(*)$

$$\Longrightarrow \qquad 2 \cdot 400 \cdot 10 = 2 \cdot 500 \cdot y'$$

$$\Rightarrow \qquad y' = \frac{\cancel{2} \times \cancel{400} \times \cancel{10}^{\,2}}{\cancel{2} \times \cancel{500}} = 8 \ \text{cm/sec}$$

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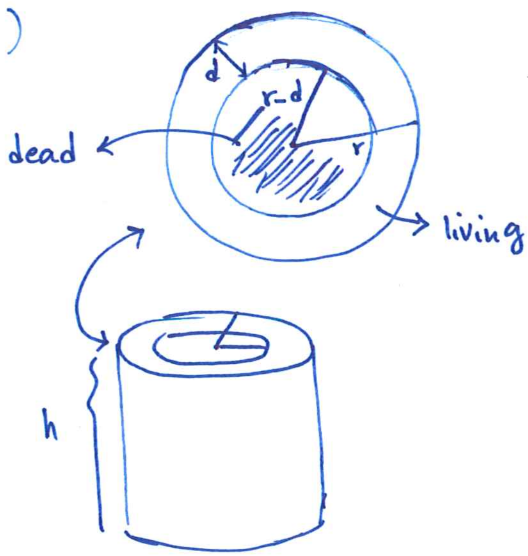
3)



a) Changing quantities:

radius of trunk $= r$

height of the tree $= h$

b) Volume of the living the tissue

= Volume of the shell in between

= Volume of trunk − Volume of dead part

Volume of tree trunk $= \pi r^2 h$, volume of the dead part $= \pi (r-d)^2 h$

$\Rightarrow$ Volume of the living tissue $= V = \pi r^2 h - \pi (r-d)^2 h$

as a function of time : $V(t) \overset{\text{factor}}{=} \pi h \left(r^2 - (r-d)^2\right)$

c) To find the fraction of living tissue, we just need to divide its volume by the volume of all tree trunk

$$\Rightarrow \quad F(t) = \frac{\pi h \left(r^2 - (r-d)^2\right)}{\pi r^2 h}$$

d) We need to differentiate both sides with respect to time, but first we simplify F :

$$F = \frac{\cancel{\pi h} \left(r^2 - (r^2 - 2rd + d^2)\right)}{\cancel{\pi} r^2 \cancel{h}} = \frac{\cancel{r^2} - \cancel{r^2} + 2rd - d^2}{r^2}$$

Constant

$$\Rightarrow \quad F(t) = \frac{2r(t)d - d^2}{r^2(t)}$$

2 cm/mon

Quotient $\Rightarrow$ Rule

$$F'(t) = \frac{2d\, r'(t) \cdot r^2(t) - 2r(t) r'(t) \cdot (2r(t)d - d^2)}{r^4(t)}$$

2

What's r ? We are at an instant where $r = 5d$ and d is some fixed number so plug in into F':

$$F'_{(t)} = \frac{2d \times 2 \times (5d)^2 - 2 \times 5d \times 2 \times (2 \times 5d \times d - d^2)}{(5d)^4}$$

$$= \frac{100d^3 - 20d(\overbrace{10d^2 - d^2}^{9d^2})}{625d^4}$$

$$= \frac{100d^3 - 180d^3}{625d^4} = \frac{-80d^3}{625d^4} = \frac{-80}{625d} \ \text{cm/month}$$

The fraction of the trunk that's living tissue is decreasing.