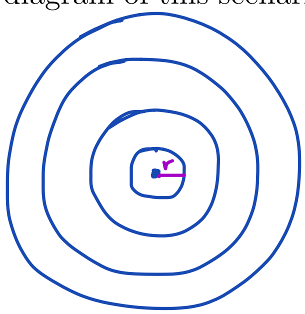
WORKSHEET: **Problem Solving Strategy in Related Rates**
MATH 110, Wednesday, Jan 10

Answer the questions following each scenario.

1. You walk alongside a calm lake and you throw a rock into the lake. Since the lake is calm, ripples in the shape of concentric circles are formed on the water.

   (a) Draw a diagram of this scenario and determine what quantities are changing over time.



As the ripples growing in size, its $\underset{r}{\underline{\text{radius}}}$ (or $\underset{d}{\underline{\text{diameter}}}$), $\underset{A}{\underline{\text{area}}}$ and $\underset{C}{\underline{\text{circumference}}}$ is increasing over time.

   (b) Assign variables to the quantities that are changing (found in part a).

   (c) Set up an equation relating area and radius of each ripple.

$$A = \pi r^2 \xrightarrow[\text{functions of time.}]{\text{Both } A \text{ and } r \text{ are implicitly}} A(t) = \pi\big(r(t)\big)^2 = \pi r^2(t)$$

→ If the radius of a ripple is increasing at a rate of 3 inches per second. Find the rate of increase in the area of the ripple when the radius is 6 inches.

unknown: $\frac{dA}{dt}$ or $A'(t)$

One instant in time when $r = 6$ inch.

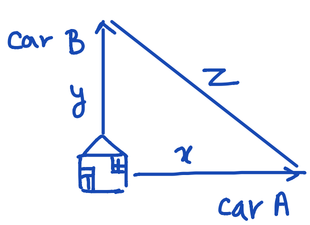
Change in the radius with respect to time: $\frac{dr}{dt}$ or $r'(t)$

$$\xrightarrow[\text{sides}]{\text{Derive both}} \big(A(t)\big)' = \big(\pi r^2(t)\big)'$$

$$\Longrightarrow A'(t) = \pi \underbrace{2r(t)\cdot r'(t)}_{\text{impliait diff}} \Longrightarrow \boxed{A'(t) = \pi \cdot 2 \cdot 6 \cdot 3 = 36\pi \ \frac{\text{inch}^2}{\text{sec}}}$$

2. Two cars start their trip from the same house. Car A is traveling east and car B is going north.

   (a) Draw a diagram of this scenario and assign variables to the distances that are changing.



These are all increasing over time. ← x : distance of car A from house
y : " " " B " "
z : distance between two cars

   (b) Set up an equation relating the assigned variables.

$$x^2 + y^2 = z^2 \xrightarrow[\text{functions of time}]{\text{all distances are implicitly}} \big(x(t)\big)^2 + \big(y(t)\big)^2 = \big(z(t)\big)^2$$

change in distance x: $\frac{dx}{dt}$ or $x'(t)$ $\qquad$ $\frac{dy}{dt} = y'(t)$

→ If car A is traveling at a speed of $30\,km/h$ and car B is traveling at a speed of $45\,km/h$, at what rate is the distance between the two cars increasing when car A is $6\,km$ and car B is $8\,km$ away from the house.

unknown: $\frac{dz}{dt} = z'(t)$

Derive both sides of above, use implicit diff →

$$2xx' + 2yy' = 2zz'$$

$$2 \cdot 6 \cdot 30 + 2 \cdot 8 \cdot 45 = 2 \cdot 10 \cdot z'$$

$$360 + 720 = 20z'$$

$$\frac{1080}{20} = z' \Rightarrow \boxed{z' = 54 \frac{km}{h}}$$

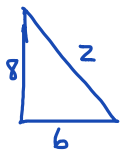
one moment in time when $x = 6$ and $y = 8$

what is z? → Pythagoras

$$6^2 + 8^2 = z^2$$

$$100 = z^2$$

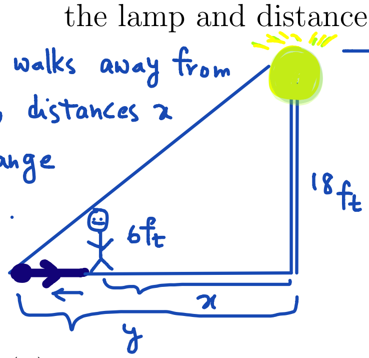
$$10 = \sqrt{100} = z$$



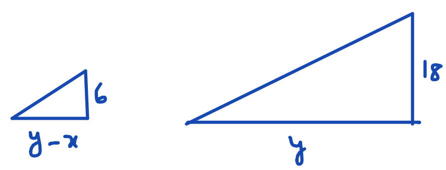
3. A person 6 feet tall walks away from a streetlamp 18 feet above ground level. The light at the top of the lamp casts a shadow in front of the person.

(a) Draw a diagram of this situation. Assign variables to the distance of the person from the lamp and distance of the "tip" of his/her shadow from the lamp. x $\quad$ y

* The person walks away from the lamp, distances x and y change over time.



We have two similar triangles:



(b) Set up an equation that relates the variables in part (a).

$$\frac{\cancel{18}\,3}{\cancel{6}} = \frac{y}{y-x}$$

$$\frac{3}{1} = \frac{y}{y-x}$$

We can also write the ratios as

$$\frac{6}{18} = \frac{y-x}{y} \quad \text{or} \quad \frac{6}{y-x} = \frac{18}{y}$$

change in x : $\frac{dx}{dt} = x'(t)$

→ If this person walks away from the lamp at a rate of $5\,ft/s$. How fast is the "tip" of the shadow moving along the ground?

unknown: $\frac{dy}{dt} = y'(t)$

$$3(y-x) = y$$

$$3y - 3x = y$$

$$3y - y = 3x$$

relation between x and y → $2y = 3x$

x and y are functions of time implicitly →

$$2y(t) = 3x(t)$$

Derive both sides →

$$2y'(t) = 3x'(t)$$

$$2y'(t) = 3 \cdot 5$$

$$\boxed{y'(t) = \frac{15}{2}\,ft/s = 7.5\,ft/s}$$