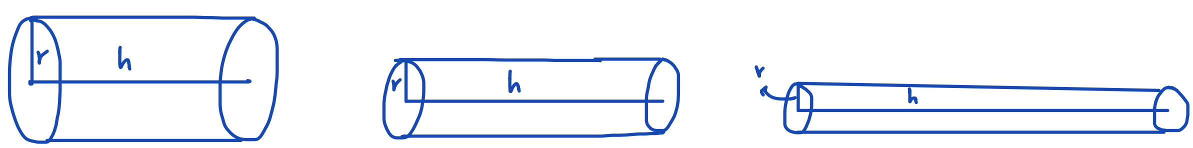
WORKSHEET 3: **Related Rates** Jan 17, 2018

1. A baker rolls a dough that has a circular cylinder form. As he rolls, the shape remains a cylinder and it becomes thinner such that the radius of the dough is decreasing at a rate of $0.5\, cm/s$. How fast is the length increasing when radius is $10\, cm$ and length is $25\, cm$?

a) Draw a picture of the situation at different times. (draw for 2 or 3 instants in time)



b) Choose one of your pictures and based on the given solution, assign variables to the changing quantities. radius = r , length (height) = h

c) Answer the questions about the given solution.

Solution:

$V = \pi r^2 h$

What's this formula? Volume of a cylinder with radius r and height h.

How would it help us? It's relating the varibles r and h.

what are r and h?

radius

height or length

$0 = \pi(\boxed{2\, r\, r'} h + r^2 h')$

why 0? Because volume remains fixed → derivative = 0

Where did this come from? Applying the product rule on $r^2 h$.

Where did this come from? This is derivative of $(r(t))^2$ by implicite differentiation.

$0 = -\pi \times 2 \times 10 \times 0.5 \times 25 + \pi \times 100 \times h' \qquad \Rightarrow h' = 2.5\, cm/sec$

Why negative? because $r'(t) = -0.5$ Since r is decreasing.

Which quantities are they?

10 = r

−0.5 = r′

25 = h

100 = r^2

Do the computation ...

d) How this question is different from Question 3 in worksheet #2?

In Q3, worksheet # 2, Volume is changing while the height is increasing but the radius is fixed. In this example, radius and height are changing but the volume remains fixed.

⋆ Include the units in your answer.

2. A rocket is launched on a vertical trajectory and is tracked by a radar station that is 3 km from the launch pad. $\frac{dx}{dt} = x'(t)$ ⤳ unknown

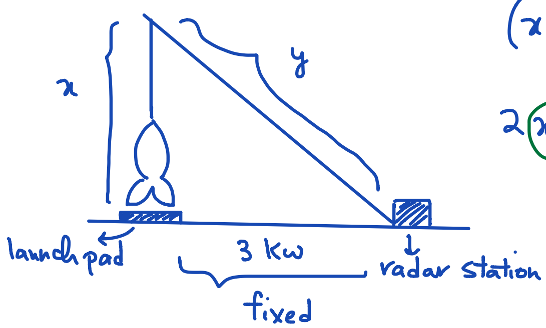
a) Find the vertical speed of the rocket at the instant when the distance from the rocket to the radar station is 5 km and that distance is increasing at 5000 km/h.

substitute it later ← $y = 5$ $\qquad dy/dt = y'(t)$

b) If the radar station is always kept aimed at the rocket, how fast is the angle of elevation changing at that same moment? $d\theta/dt = \theta'(t)$ ⤳ unknown

(Hint: The angle of elevation is the angle that the line of sight in the radar station makes with the horizontal axis.)

a)



$$(x(t))^2 + 3^2 = (y(t))^2$$

$$2\,x(t)\,x'(t) + 0 = 2\,y(t)\,y'(t)$$

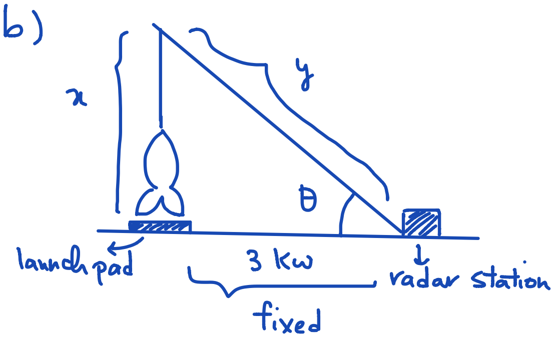
$x'(t)$: unknown, $y(t)$: 5, $y'(t)$: 5000

$$\boxed{\begin{array}{l}\text{Use Pythagorean Thm to find } x \text{ at a moment:}\\ x^2 + 3^2 = 5^2\\ \Rightarrow x^2 = 25 - 9 = 16\\ \Rightarrow x = 4\end{array}}$$

$$2 \cdot 4 \cdot x' = 2 \cdot 5 \cdot 5000$$

$$\Rightarrow x' = \frac{\cancel{2} \times 5 \times 5000}{\cancel{2} \times 4} = \frac{25000}{4} \; km/h$$

(b)



Use any of the trig ratios :

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}, \quad \cos\theta = \frac{\text{adj}}{\text{hyp}}, \quad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

Let's see which one is the best choice :

$$\underbrace{\sin\theta = \frac{x}{y}}_{\text{The hardest because we need to apply quotient rule}}, \quad \underbrace{\cos\theta = \frac{3}{y}}_{\text{easier : } 3y^{-1}}, \quad \underbrace{\tan\theta = \frac{x}{3}}_{\text{easiest : } \frac{1}{3}x}$$

↳ power rule $\qquad$ ↳ simple derivative

$$\underbrace{\tan}_{\text{outside}}\ \underbrace{\theta(t)}_{\text{inside}} = \frac{x(t)}{3}$$

Derive $\rightarrow$ $\underbrace{\left(1+\tan^2\theta(t)\right)}_{\text{derivative of } \tan} \cdot \theta'(t) = \frac{1}{3}\, x'(t)$

$\theta'(t)$ $\downarrow$ unknown

$x'(t)$ in part (a) calculated $\longrightarrow$ If there's no part (a) then $\cos\theta = \frac{3}{y}$ is the best choice because y' is given.

$$\boxed{\begin{array}{c}\text{Calculate } \tan\theta = \frac{x}{y} \\ \text{at the given moment} \\ x = 4\ ,\ y = 3 \\ \tan\theta = \frac{4}{3}\end{array}}$$

$$\left(1+\left(\cancel{\tfrac{4}{3}}\right)^2\right) \cdot \theta'(t) = \frac{1}{3} \cdot \frac{25000}{4} \qquad \left(\tfrac{4}{3}\right)^2 = \frac{16}{9}$$

$$\frac{25}{9}\,\theta'(t) = \frac{25000}{12}$$

$$\theta'(t) = \frac{\cancel{25000}}{\cancel{12}_{\,4}} \cdot \frac{\cancel{9}^{\,3}}{\cancel{25}} = \frac{3000}{4}\ \text{rad}/\text{h}$$