Homework 3, MATH 110-001

Due date: Friday, Feb 9, 2018 (in class)

Hand in full solutions to the questions below. Make sure you justify all your work and include complete arguments and explanations. Your answers must be clear and neatly written, as well as legible (no tiny drawings or micro-handwriting please!). Your answers must be stapled, with your name and student number at the top of each page.

1. Suppose two sprinters racing each other finish in a tie. Explain, using the Mean Value Theorem, why this means there must have been a moment in the race when the two sprinters were running at exactly the same speed.

2. Find the local extrema of the following functions by using the first derivative test. For part (a) also determine the inflection point and the concavity of the function. (You do not need to calculate the y-coordinates for local extrema and the inflection points.)

   (a) $f(x) = \dfrac{x}{2} - \arctan(x)$

   Hint: Note that $\arctan(x)$ is the inverse function for $\tan(x)$ i.e. $\arctan(\tan(x)) = x$, and its derivative is given by $(\arctan(x))' = \dfrac{1}{1+x^2}$.

   (b) $g(x) = -\dfrac{1}{5}(x-4)^{\frac{5}{3}} - 2(x-4)^{\frac{2}{3}}$

3. Find the critical numbers of the implicit function defined by the equation

$$x^2 + y^2 - 3xy + 5 = 0.$$

   You do not need to find the local extrema.

4. Sketch a graph for the function f that satisfies all the following conditions.

   (i) Domain: $[0, 6]$

   (ii) x-intercept $= 6$, y-intercept $= 4$

   (iii) $f'(x) < 0$ when $x < 2$ and $x > 4$

   (iv) $f'(2)$ does not exist.

   (v) $f'(4) = 0$

   (vi) $f'(x) > 0$ on $[2, 4]$

   (vii) $f''(x) > 0$ for all x in the domain