

Homework 5, MATH 110-001

Due date: Thursday, March 29, 2018

in office hour, 2-3pm, LSK 300

Hand in full solutions to the questions below. Make sure you justify all your work and include complete arguments and explanations. Your answers must be clear and neatly written, as well as legible (no tiny drawings or micro-handwriting please!). Your answers must be stapled, with your name and student number at the top of each page.

In the following, take great care when setting up the problems. Use the following steps as a guide.

- Understand the problem. Read the problem carefully. Try to draw a picture or think about some examples. Get a feel for what the relevant quantities are, how the quantities can change and what constraints are present.
- Introduce notation for the relevant quantities. Identify the quantity to be optimized (objective function).
- Find relationships between the variables (constraint(s)).
- Find an equation for the quantity to be optimized as a function of a single variable on some suitable domain.
- Do Calculus. Use techniques from the course to find the absolute maximum or minimum as required. For example, if you are working with a continuous function on a closed interval consider using the closed interval test.
- Reflect. Take some time to think about your answer. Is it reasonable?

Questions

1. Your love has been lost at sea for weeks. Each day you stand on a cliff, at point $(0, 0)$, looking out to sea wondering if they will return. One day you see a boat due north at point $(0, 2)$. You notice the boat is traveling down the river along the curve $y = \sqrt{x+4}$ toward the harbour, which is west of you at point $(-4, 0)$. When the boat reaches the point where it is closest to your position you recognize it as your love's boat. Find the coordinates of this point and find your distance from your love?

2. You are selling subscriptions to an online magazine. Based on some market research you determine that if you set the price of the subscription to be p in dollars then $Q(p)$ is the quantity of subscriptions you are able to sell (in thousands) where

$$Q(p) = 6 - 2\sqrt{p}$$

The revenue you will generate is given by

$$R(p) = \text{price} \cdot \text{quantity} = p \cdot Q(p)$$

What price should you set in order to maximize revenue? What will be your maximum revenue?

3. Find the area of the largest rectangle that can be inscribed in a right triangle with legs of lengths a cm and b cm if two sides of the rectangle lie along the legs.
4. A right triangle of hypotenuse D cm is revolved about one of its legs to create a right circular cone. Find the radius, height, and volume of the cone of the greatest volume.
(Volume of a cone = $\frac{1}{3}$ (area of its base) $\cdot$ height)

