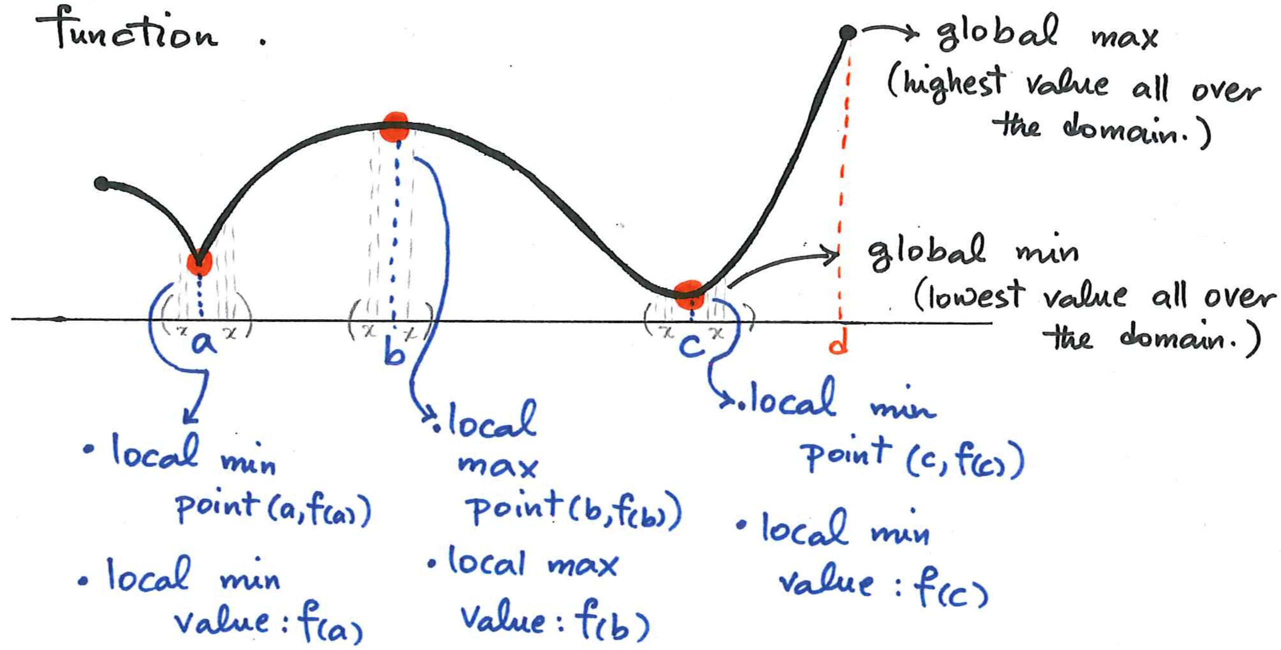
Optimization

Lecture 26
March 12

First, let's review the local extrema and then introduce the global extrema of a function.

<u>Example</u>. Find the local extrema of the following function.



Recall the formal definition:

<u>Definition</u>. Function f has a local minimum at "a" if for all x's in an interval around "a" (that contains "a") we have: $f(a) < f(x)$.

the smallest value among all x's.

Similarly;

f has a local maximum at "a" if for all x's in an interval around "a" (that contains "a") we have: $f(a) > f(x)$.

↳ the largest value among all x's.

Now go back to the previous graph and find the global min/max of the given function.
or absolute

global vs. local ? global extrema is the highest/lowest value all over the domain whereas local min/max is only considered in some neighborhood or interval.

Some more formal terminology :

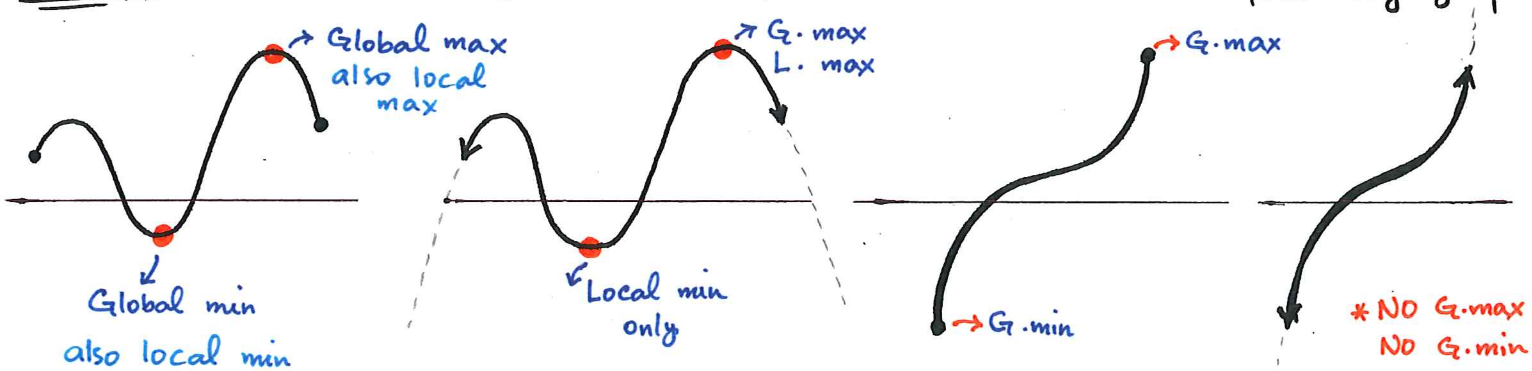
Definition (global minimum) Function f has a global minimum at "a" , if for all x's over the whole domain of f , we have $f(a) < f(x)$.

x and y
global min point : $(a, f(a))$
global min value : $f(a)$
y-value

In the previous graph:
G. min point : $(c, f(c))$
G. min value : $f(c)$
G. max point : $(d, f(d))$
G. max value : $f(d)$

Exercise . Write the definition of global maximum .

Example . Find the global extrema in each of the following graph



* Global max & min occur at local max & min .

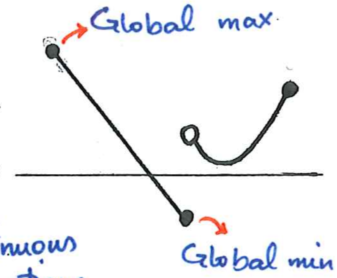
* No G. min , because the domain is not a closed interval , Domain : $(-\infty, \infty)$

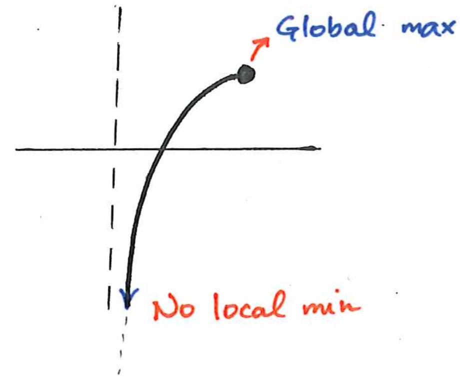
* Global extrema occur at endpoints .

* NO local extrema

What about ?

What's different about these graphs comparing to the previous ones? discontinuous functions.





Important Facts

Square brackets means a & b both are included in the domain.

- If f is continuous on a closed interval [a,b], then f has both a global max and a global min on the interval.

- If f is continuous on a closed interval [a,b], the global max/min occur either at an endpoint or at a local max/min point.

How to find global max/min of a function without graphing?

We are sure there is a G.min and G. max

→ $\boxed{\text{f continuous and a closed } [a,b] \text{ is given}}$ → Find all critical numbers and evaluate the function value for them.

Also evaluate f(a) and f(b).

⇝ Compare all the values

⇝ Smallest: global min, largest: global max

<u>Example</u>: Find global extrema of $y = x^2$ on $[-2, 3]$.

Find Global extrema of $y = x^2$ on the interval $[-2, 3]$.

$y = x^2$ is a cont's function on the closed interval $[-2, 3]$, so the global extrema occurs either at the endpoints or at the local extrema. So the first step is to find the local extrema like what we did in previous weeks.

$y = x^2 \longrightarrow y' = 2x$ → everywhere defined

→ $2x = 0 \Rightarrow x = 0$

	test −1		test 1	
x	$-\infty$	0		∞
f'	$-$		$+$	
f	↘	local min	↗	

Now check the y-coordinates and compare

local extrema $\xrightarrow[\text{min}]{\text{local}}$ $x = 0 \Rightarrow y = (0)^2 = 0$ ⇝ smallest

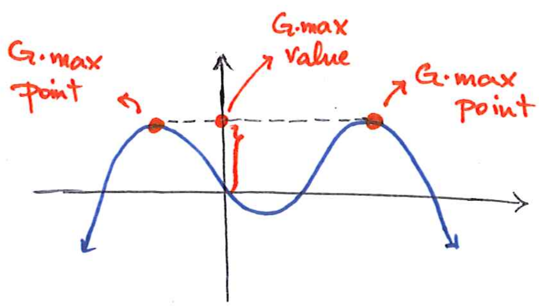
G. min value $= 0$
G. min point $= (0, 0)$

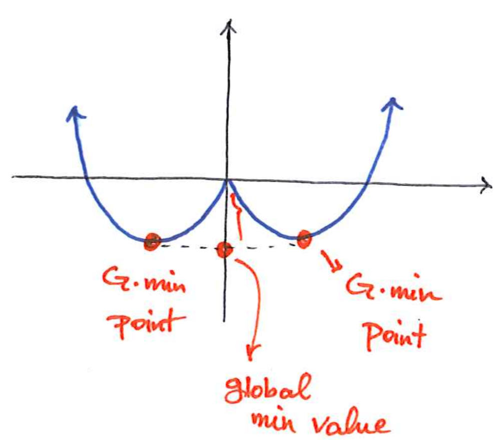
endpoints ⇝ $x = -2 \Rightarrow y = (-2)^2 = 4$

$x = 3 \Rightarrow y = (3)^2 = 9$ ⇝ largest

G. max value $= 9$
G. max point $= (3, 9)$

⊛ Note. A function can have multiple global extrema points BUT it can have only one global extrema value.





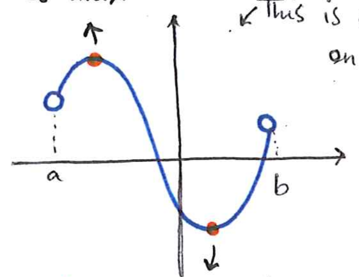
?? Why isn't it possible to have multiple global extrema values?

?? What can we say about # of local extrema points / values?

→ $\boxed{f \text{ continuous BUT NO closed interval is given}}$ → It differs case by case. Both global min/max, NO global min/max or one but not the other.

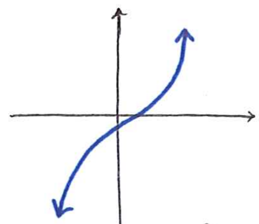
G· max

G· min.



This is a continuous function on (a,b).

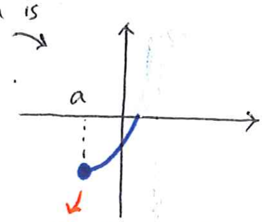
Both G·min & G·max occur.



NO G·min & G·max.

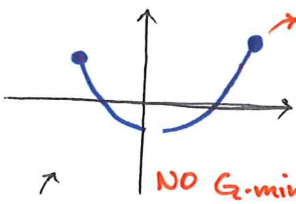
This is a cont's function on $(-\infty,\infty)$.

This function is continuous on (a,∞).



G·min, NO G·max

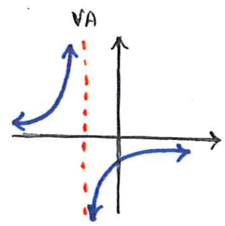
→ $\boxed{f \text{ NOT continuous}}$ → Again, must be checked case by case. We might have asymptotes or holes or jumps.



G·max

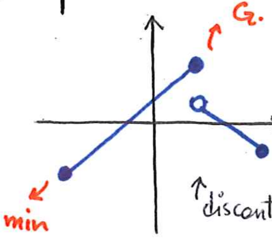
NO G·min

discont's because of hole.



discontinuous because of the VA. (undefined)

NO G·max & G·min



G. max

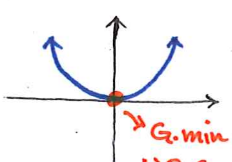
G·min

discont's : jump

Exercise. For the well-known functions that you are familiar with, verify whether they have global min/max and if so find them.

You should remember all these graphs and all their properties.

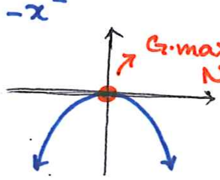
$y = x^2$



G·min

NO G·max

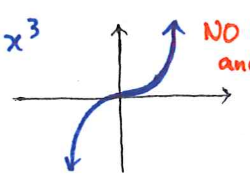
$y = -x^2$



G·max

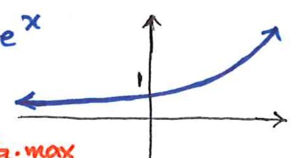
NO G·min

$y = x^3$



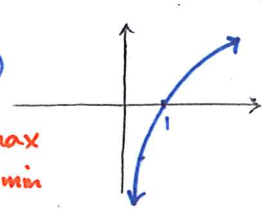
NO G·max and G·min

$y = e^x$



NO G·max, NO G·min

$y = \ln(x)$

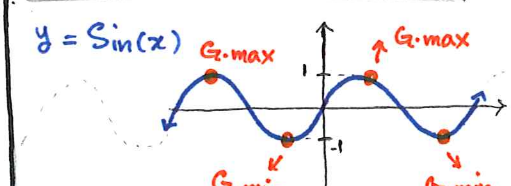


NO G·max and G·min

Oscillating functions: multiple G·max and G·min.

G·max value $=1$, G·min value $=-1$

$y = \mathrm{Sin}(x)$



$y = \mathrm{Cos}\,x$