March 21 → Happy Spring :)

Reminder : • Quiz 5 on Friday, on Optimization.

• this week, NO office hour tomorrow

Instead : Friday, 3-4 pm

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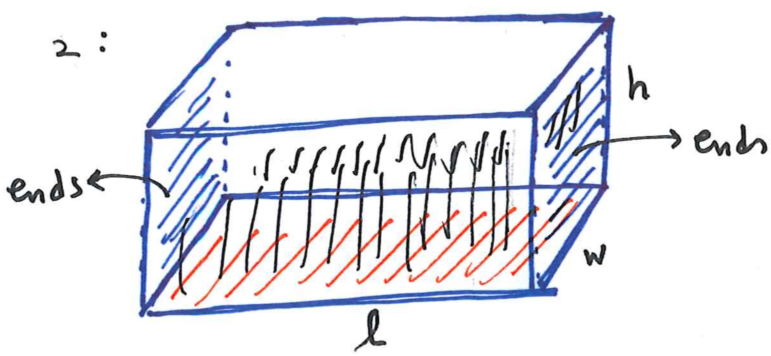
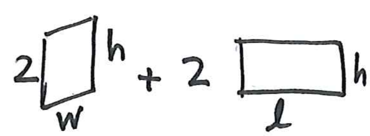
4. (worksheet)

Step 1 : Minimizing the cost.

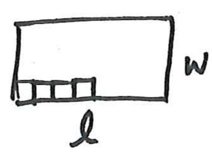
2 :



3 : total cost = Cost of bottom + Cost of sides 2



5\$



+

$= 5\,(\text{Area (bottom)}) + 20\,(\text{area of two ends} + \text{area of front and back})$

$$C = 5(lw) + 20(2hw + 2lh)$$

↓

3-variables.

4 : Constraint :

$$\begin{cases} w = \frac{1}{4} l \Rightarrow l = 4w \\ \text{Volume} = 100 \Longrightarrow lwh = 100 \Rightarrow h = \frac{100}{lw} = \frac{100}{4w^2} = \frac{25}{w^2} \end{cases}$$

$4w \leftarrow l$

5 : Use constraints to reduce total C ~~as~~ to a function one variable. $\left(h = \frac{25}{w^2},\ l = 4w\right)$ ✓

$$C = 5(4w \cdot w) + 20(2hw + 2(4w)h) \rightarrow \text{2-variables}$$

$$C = 5(4w^2) + 20\left(2 \cdot \frac{25}{w^{\cancel{2}}} \cdot \cancel{w} + 8\cancel{w}\frac{25}{w^{\cancel{2}}}\right)$$

$$C = 20w^2 + 20\left(\frac{50}{w} + \frac{200}{w}\right)$$

$$\longrightarrow C(w) = 20\left(w^2 + \frac{250}{w}\right)$$

6 : Domain : $w > 0$

7 : Calculus : $C'(w) = 20\left(2w + 250 \cdot \frac{-1}{w^2}\right)$

$$C'(w) = 20\left(2w - \frac{250}{w^2}\right)$$

$C'(w) = 0$

$$2w - \frac{250}{w^2} = 0$$

undefined at $w = 0$ but 0 not in the domain.

$$2w - \frac{250}{w^2} = 0 \quad \boxed{\rightsquigarrow \text{ or } \quad 2w = \frac{250}{w^2}}$$

$$\frac{2w^3 - 250}{w^2} = 0 \Rightarrow 2w^3 - 250 = 0$$

$$\Rightarrow w^3 = \frac{250}{2} = 125$$

$$\Rightarrow w = \sqrt[3]{125} = 5$$

Final checks:

→ $\cancel{w} = 5$ a local min ?

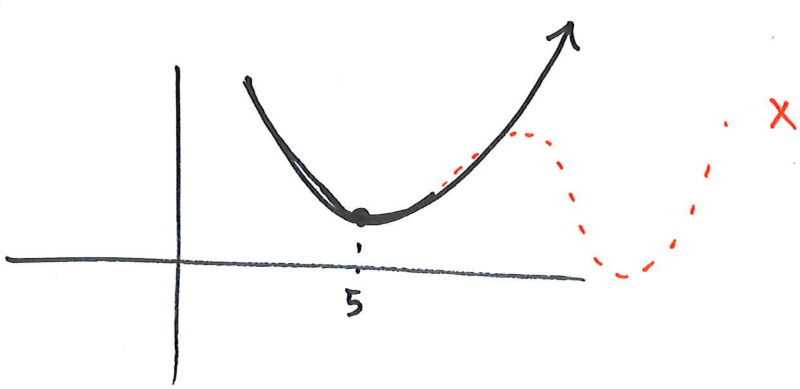
2nd derivative test: $C''(w) = 20\left(2 - 250 \cdot \frac{-2}{w^3}\right)$

$$= 20\left(2 + \frac{500}{w^3}\right)$$

$$\xrightarrow{x=5} C''(5) > 0$$ ∪ ↘ local min.

→ $\cancel{w} = 5$ a global min ?

open interval $(0, \infty)$ ⇝ roughly visualize the graph of C



$C''(w)$ is always ⊕ for $w > 0$ ⇒ the function is always concave up.

⇒ local min is the global min.

Dimensions:

$w = 5$ inch

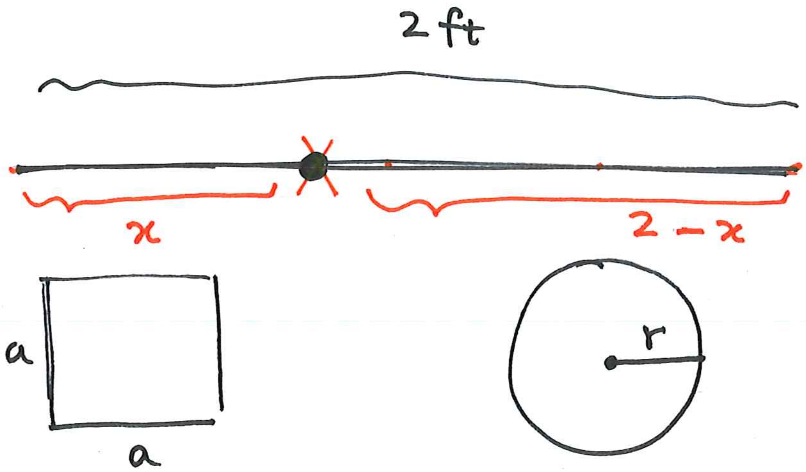
$l = 4w = 4 \cdot 5 = 20$ inch

$h = \frac{25}{w^2} = \frac{25}{25} = 1$ inch

Cheapest box. ←

Q3. (worksheet).

2)



1) Maximizing the area of the square + the area of the circle.

3) formulate the area:

$$\text{total area} = a^2 + \pi r^2$$

4). Constraint: 2 ft of wire.

x is the perimeter of the square.

$2 - x$ is the circumference of the circle.

$$\begin{cases} x = 4a & \Rightarrow \boxed{\dfrac{x}{4} = a} \\ 2-x = 2\pi r & \Rightarrow \boxed{\dfrac{2-x}{2\pi} = r} \end{cases}$$

total area : $\left(\dfrac{x}{4}\right)^2 + \pi\left(\dfrac{2-x}{2\pi}\right)^2$

$$A = \frac{x^2}{16} + \cancel{\pi}\,\frac{(2-x)^2}{4\pi^{\cancel{2}}}$$

$$A(x) = \frac{x^2}{16} + \frac{(2-x)^2}{4\pi}$$

Domain : $0 \leq x \leq 2$

all one circle

all one square.

Exercise. Complete the example.