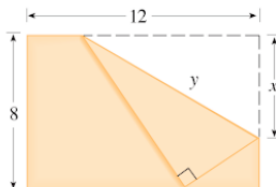


1. **Lifeguard Problem.**

You are a lifeguard at a beach in Vancouver. One day, as you are sitting on your lifeguard chair, you see a swimmer calling for help. The swimmer appears to be roughly 120 feet out to sea (on a straight line between the swimmer and the shore), and the lifeguard station is roughly 300 feet down the beach from the nearest point on shore to the swimmer. You can run at 13 feet per second along the beach, and you can swim at 5 feet per second. At what point along the shoreline should you enter the water so as to reach the swimmer as quickly as possible? How far do you run and how far do you swim?

2. We want to make a cylindrical can with volume 128 in^3 by cutting its top and bottom from squares of metal and forming its curved side by bending a rectangular sheet of metal to match its ends. What radius r and height h of the can will minimize the total amount of material required for the rectangle and the two squares? How would it be different if it is an open top can?
3. A 2 feet piece of wire is cut into two pieces and one piece is bent into a square and the other is bent into a circle. Where should the wire be cut so that the total area enclosed by both is maximum?
4. You want to make a box with an open top such that the width of its base is one fourth of its length. If the material to build the bottom is 5\$ per square inch and the material for the sides costs 20\$ per square inch. What are the dimensions of the cheapest box which will hold 100 cubic inches of water?
5. The upper right-hand corner of a piece of paper, 12 in. by 8 in. as in the figure, is folded over to the bottom edge. How would you fold it so as to minimize the length of the fold? In other words, how would you choose x to minimize y ?



6. Find the points on the parabola $y = x^2 + 1$ that are closest to $(0, 2)$.
7. Of all lines tangent to the graph of $y = \frac{6}{x^2 + 3}$, find the tangent lines of minimum slope and maximum slope.
8. What is the area of the largest triangle that can be formed in the first quadrant by the x -axis, the y -axis and a tangent line to the graph $y = e^{-x}$?
9. (a) A right circular cylinder is inscribed in a cone with height 6 m and base radius 3 m. Find the largest possible volume of such a cylinder.
(b) Find the largest volume of a cylinder as in part (a) for a general right circular cone with height h and base radius r .
10. The following equation represents the power P that is required to keep a plane moving at a speed v ,

$$P(v) = av^2 + \frac{b}{v^2}$$

where a and b are positive constants. Determine the speed at which the least power is required to keep the plane moving.

11. Find the area of the largest rectangle that can be inscribed in a right triangle with legs of lengths a cm and b cm if two sides of the rectangle lie along the legs.
12. Car B is 30 miles directly east of Car A and begins moving west at v_B mph. At the same moment car A begins moving north at v_A mph. At what time are the two cars closest?