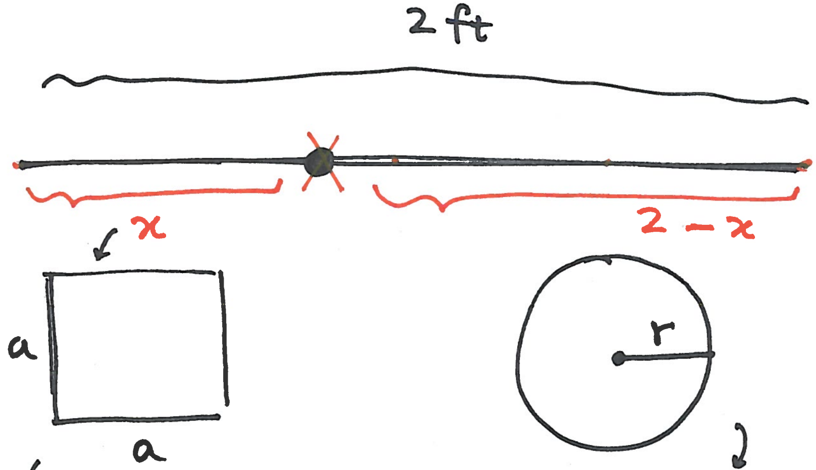
Q3. (worksheet).

2)



1) Maximizing the area of the square + the area of the circle.

x feet becomes the square.

$2-x$ feet becomes the circle.

3) formulate the area:

$$\text{total area} = a^2 + \pi r^2$$

4). Constraint: 2 ft of wire.

x is the perimeter of the square.

$2-x$ is the circumference of the circle.

$$\begin{cases} x = 4a & \Longrightarrow \quad \boxed{\dfrac{x}{4} = a} \\[2ex] 2 - x = 2\pi r & \Longrightarrow \quad \boxed{\dfrac{2-x}{2\pi} = r} \end{cases}$$

total area : $\left(\dfrac{x}{4}\right)^2 + \pi \left(\dfrac{2-x}{2\pi}\right)^2$

$$A = \frac{x^2}{16} + \cancel{\pi}\,\frac{(2-x)^2}{4\pi^{\cancel{2}}}$$

$$A(x) = \frac{x^2}{16} + \frac{(2-x)^2}{4\pi}$$

Domain: $0 \leq x \leq 2$

all one circle (for $x = 0$)

all one square (for $x = 2$).

Exercise. Complete the example. (Solution: a separate link)

$$A(x) = \frac{x^2}{16} + \frac{(2-x)^2}{4\pi}, \qquad 0 \le x \le 2$$

We want to maximize A. (Global max)

$$A'(x) = \frac{1}{\cancel{16}_{8}}(\cancel{2}x) + \frac{1}{\cancel{4\pi}_{2}}\underbrace{(-1)}_{\text{inside}}\underbrace{(2-x)(\cancel{2})}_{\text{outside}}$$

$$\Rightarrow A'(x) = \frac{1}{8}x - \frac{1}{2\pi}(2-x)$$

$$A'(x) = 0 \Rightarrow \frac{\pi x - 4(2-x)}{8\pi} = 0$$

$$\Rightarrow \pi x - 8 + 4x = 0 \Rightarrow (\pi + 4)x = 8$$

$$\Rightarrow x = \frac{8}{\pi+4} \approx 1.12$$

- Check if $x = \frac{8}{\pi+4}$ is a local max ?

$$\xrightarrow[\text{test}]{2^{nd}\text{ derivative}} A''(x) = \frac{1}{8} + \frac{1}{2\pi} > 0 \xrightarrow[\text{up}]{\text{Concave}}$$

$\rightsquigarrow$ This critical number is a local min NOT a local max !!!

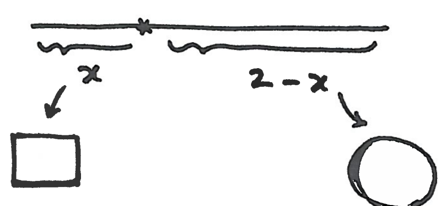
In fact if we compare the area at $x = \frac{8}{\pi+4}$ then $A\left(\frac{8}{\pi+4}\right) \approx 0.14 \rightsquigarrow$ smallest area.

However, at the end points:

$$A(0) = 0 + \frac{(2-0)^2}{4\pi} = \frac{4}{4\pi} \approx 0.32 \rightsquigarrow \boxed{\text{largest area}} \xrightarrow{x=0}$$

Recall the diagram:

$$A(2) = \frac{2^2}{16} + \frac{(2-2)^2}{4\pi} = \frac{1}{4} = 0.25$$



When $x=0$, there will be NO square.

If we want to maximize the area. All the wire should become a circle