MATH 110-001, QUIZ 5

March 23, 2018

Time: 15 minutes

Show all your work. No calculators, no books/notes are allowed.

Name (please print): total = 10

Student number: Solution.

8 pts

1. Find two positive numbers such that the sum of the first and twice the second is 80 and their product is maximum. → Objective function: Product of x and y

Constraint: $x + 2y = 80 \Rightarrow (x = 80 - 2y)$

objective: $P = xy = (80 - 2y)y$

So $P(y) = 80y - 2y^2$, $y > 0$

$P'(y) = 80 - 4y \xrightarrow{P'(y)=0} 80 - 4y = 0$

$\Rightarrow y = \frac{80}{4} = 20 \rightsquigarrow$ local max ?

2nd derivative test: $P''(y) = -4 < 0$

Is it global max? This function is always concave down so local max is the global max.

(Otherwise, the concavity will change and contradicts.)

Alternatively: $y = \frac{80 - x}{2} = 40 - \frac{x}{2}$

$P = xy = x(40 - \frac{x}{2}) = 40x - \frac{1}{2}x^2$

$\Rightarrow P'(x) = 40 - x = 0$

$\Rightarrow x = 40 \rightsquigarrow$ local max ?

$P''(x) = -1 < 0$

The same reasoning for global max.

Find the other number:

$x = 40 \Rightarrow y = 40 - \frac{x}{2} = 20$

Find the other number

$y = 20 \Rightarrow x = 80 - 2y = 80 - 40 = 40$

2 pts

2. Function f is continuous, differentiable and everywhere concave down and it has a critical point at $x = 1$. Choose all the correct options.

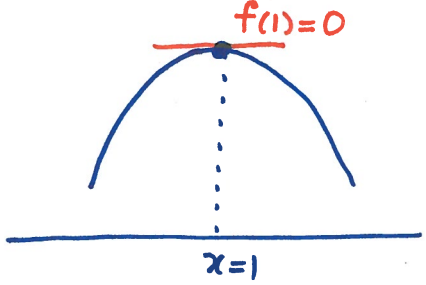
(a) f has a local minimum at $x = 1$

(b) f has a local maximum at $x = 1$

(c) f has a global minimum at $x = 1$

(d) f has a global maximum at $x = 1$

(e) f has an inflection point at $x = 1$



Both a local max & a global max.

* Always concave down ⇝ NO change in concavity ⇝ NO INF point.