Solution to the curve sketching exercise :

Sketch the function $y = x^4 - 4x^3$

$\boxed{f \text{ itself}}$: Domain $= \mathbb{R}$

intercepts $= \begin{cases} y\text{-int} \xrightarrow{x=0} y = 0^4 - 4 \cdot 0^3 = 0 \Rightarrow (0,0) \\ x\text{-int} \xrightarrow{y=0} 0 = x^4 - 4x^3 \end{cases}$

$$\xrightarrow[\text{factor}]{\text{doable}} 0 = x^3(x-4)$$

$$\Rightarrow \quad x^3 = 0 \quad \text{and} \quad x - 4 = 0$$

$$\Rightarrow \quad x = 0 \;, \; x = 4$$

$$\Rightarrow \quad (0,0) \;, \; (4,0)$$

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$\lim\limits_{x \to \infty} x^4 - 4x^3 = \infty$ and $\lim\limits_{x \to -\infty} x^4 - 4x^3 = \infty \Rightarrow f$ goes upward in both ends of x-axis.

$\boxed{f'}$: INC/DEC

$f'(x) = 4x^3 - 12x^2$ everywhere defined

$f'(x) = 0 \Rightarrow 4x^3 - 12x^2 = 0$

$\xrightarrow{\text{factor}} 4x^2(x-3) = 0$

$\rightarrow 4x^2 = 0 \Rightarrow x = 0$

$\rightarrow x - 3 = 0 \Rightarrow x = 3$

f' Sign Chart

	test -1		test 1		test 4	
x	$-\infty$	0		3		∞
f'	$-$	0	$-$	0	$+$	
f	$\searrow$		$\searrow$	local min	$\nearrow$	

$f'(x) = (4x^2)(x-3)$ — always $\oplus$

$f'(4) = \oplus(4-3) > 0$

$f'(1) = \oplus(1-3) < 0$

$f'(-1) = \oplus(-1-3) < 0$

y-coordinate :

$f(0) = 0 \;, \quad f(3) = 3^4 - 4(3)^3 = -27$

$\Rightarrow (3, -27)$ local min point .

$\boxed{f''}$:

Concavity

$$f'(x) = 4x^3 - 12x^2$$

$$f''(x) = 12x^2 - 24x \quad \text{is defined everywhere.}$$

$$f''(x) = 0 \Rightarrow 12x^2 - 24x = 0$$

$$\Rightarrow 12x(x-2) = 0 \rightarrow x = 0,\ x = 2$$

Sign chart for f''.

x	test -1	0	test 1	2	test 3
f''	$+$	0	$-$	0	$+$
f	$\cup$	INF	$\cap$	INF	$\cup$

$f''(x) = 12x(x-2)$ always $\oplus$

$f''(3) = \oplus\, 3(3-2) > 0$

$f''(1) = \oplus\, 1(1-2) < 0$

$f''(-1) = \oplus\, (-1)(-1-2) > 0$

y-coordinates : $f(2) = 2^4 - 4(2)^3 = -16$

$\Rightarrow$ $(0,0)$ and $(2,-16)$ are the two inflection points.

Summary Chart:

x	$-\infty$	0		2		3	∞
f'	$-$ ↘	0	$-$ ↘		$-$ ↘	0	$+$ ↗
f''	$+$ $\cup$	0	$-$ $\cap$	0	$+$ $\cup$		$+$ $\cup$
f	↘	INF $(0,0)$	↘	INF $(2,-16)$	↘	Min $(3,-27)$	↗