MATH 110-001, QUIZ 4

March 9, 2018

Time: 15 minutes

Show all your work. No calculators, no books/notes are allowed.

Name (please print): Solution

Student number: ____________________

1. (a) Find the following limit. (Show your work.)

$$\lim_{x\to\infty} x \sin\left(\frac{1}{x}\right)$$

$$\lim_{x\to\infty} x \sin\left(\frac{1}{x}\right) = \infty \sin\left(\frac{1}{\infty}\right) = \infty \sin(0) = \infty \times 0$$

Rewrite the function as a quotient: $\lim_{x\to\infty} \dfrac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}} = \dfrac{\sin\left(\frac{1}{\infty}\right)}{\frac{1}{\infty}} = \dfrac{\sin(0)}{0} = \dfrac{0}{0}$

Now we can l'Hôpital

$$\lim_{x\to\infty} \frac{\left(\sin\left(\frac{1}{x}\right)\right)'}{\left(\frac{1}{x}\right)'} = \lim_{x\to\infty} \frac{-\frac{1}{x^2}\cos\left(\frac{1}{x}\right)}{-\frac{1}{x^2}}$$

$$= \lim_{x\to\infty} \cos\left(\frac{1}{x}\right) = \cos\left(\frac{1}{\infty}\right) = \cos 0 = 1$$

(b) Use part (a) and determine the horizontal asymptote(s) of the function $f(x) = x \sin\left(\frac{1}{x}\right)$.

By definition and part (a): $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} x \sin\left(\frac{1}{x}\right) = 1$

$\Rightarrow y = 1$ is HA.

2. The problem $\lim_{x\to 0} \frac{3x}{2x^2+x}$ appeared on a test.

- Student A determined that the limit was an indeterminate $\frac{0}{0}$ form and applied l'Hopital's rule twice to get:

$$\lim_{x\to 0} \frac{3x^2}{2x^2+x} = \lim_{x\to 0} \frac{6x}{4x+1} = \lim_{x\to 0} \frac{6}{4} = \frac{6}{4}$$

- Student B also determined that the limit was an indeterminate $\frac{0}{0}$ form and applied l'Hopital's rule too to get:

$$\lim_{x\to 0} \frac{3x^2}{2x^2+x} = \lim_{x\to 0} \frac{6x}{4x+1} = \lim_{x\to 0} \frac{0}{0+1} = 0$$

Which student was correct? Why?

Student B. Applying l'Hôpital's rule once gives:

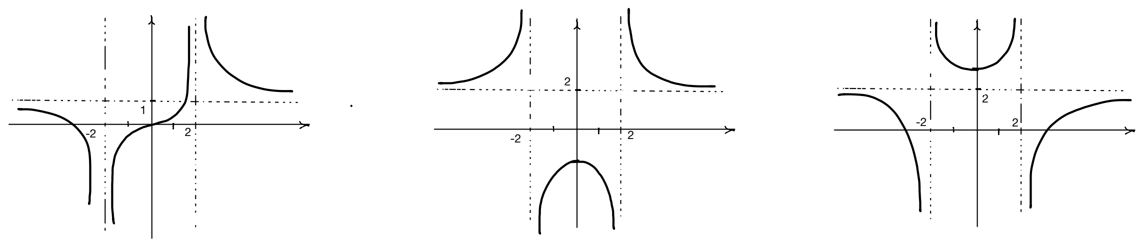
$$\lim_{x\to 0} \frac{3x^2}{2x^2+x} = \lim_{x\to 0} \frac{6x}{4x+1}$$

Now we substitute 0 we get $\frac{6(0)}{4(0)+1} = 0 \rightsquigarrow$ NO longer indeterminate.
No more l'Hôpital needed.

3. Choose the graph that matches with the function

$$f(x) = \frac{2x^2}{x^2-4}$$

and give a brief explanation for your choice. (You do NOT need to compute f' or f''.)



(A) (B) (C)

HA: $\lim_{x\to\pm\infty} \frac{2x^2}{x^2-4} = \frac{\infty}{\infty} \overset{\text{l'H}}{=} \lim_{x\to\pm\infty} \frac{4x}{2x} = 2 \Rightarrow y=2$ HA

Your explanation can be in terms of mathematical formulas.

I made a silly mistake in writing the quiz. I computed $f(0) = \frac{2(0)}{0-4} = -4$!!!
that's why if you choose (A) because $f(0)=0$, I'd take it as a correct answer because it's the only graph that passes through the origin. However, nothing else matches.

The correct was supposed to be B, because of the VA.

$\lim\limits_{x \to 2^+} \frac{2x^2}{(x-2)(x+2)} = \frac{8}{0^+} = \infty$, $\lim\limits_{x \to -2^+} \frac{2x^2}{(x-2)(x+2)} = \frac{8}{-4 \times 0^+} = -\infty$

and $\lim\limits_{x \to 2^-} f(x) = \frac{8}{0^-} = -\infty$, $\lim\limits_{x \to -2^-} f(x) = \frac{8}{-4 \times 0^-} = +\infty$