Main Steps for Optimization Problems

1. **Read the question carefully.** Look for the quantity that should be optimized. You should look for a quantity to be *maximum, minimum, largest, smallest, shortest, closest, greatest, highest, farthest ...*
2. **Draw and label a picture or diagram.** Include any information you are given in the problem, identify the quantities and assign variables to them.
3. **Make a formula for the objective (optimized) function.** This is the quantity you want to maximize or minimize. Write a formula for it in terms of the variables in your picture.
4. **Check the extra information given and make a formula for that too.** The extra information is called the *constraint* and in almost every one of the problems it is indicated as having a *fixed value.*
5. **Use constraint to reduce the objective function to a one variable function.** The constraint will give you an extra equation that relates the variables together. Solving the constraint equation for one of the variables and substitute this into the objective function will reduce the objective quantity to be a function of only one variable
6. **Find the domain of the objective function.** Make sense of the variables in the context of the problem – for example, lengths cannot be negative. Is the domain an open interval or a closed interval?
7. **Use calculus to find the critical numbers of the objective function.** These numbers are potential local maxima or minima. Ignore any critical numbers that are outside the domain of the objective function.

Extra Tips:

- *Test the critical numbers.* The critical numbers are not automatically the answer. You need to test each one to see if it is a local minimum, a local maximum, or neither.
- *Test the endpoints of the domain.* Note that the goal is to find the global max/min, so we need to verify what happens at the endpoints. In addition to evaluating the objective function at all critical numbers, find the value of the objective function at the endpoints of the domain as well, then choose the largest value (if you are maximizing) or the smallest value (if you are minimizing). If the domain is an open interval, check the limiting behavior of the function and/or its concavity.
- *Reread the problem.* Check the question to see exactly what it is asking for. If it is asking for the dimensions, for example, make sure you give them all (height, width, radius, etc.). If it is asking for the area or volume make sure you evaluate the objective function at the optimum dimensions.
- *Reflect.* Take some time to think about your answer. Is it reasonable?

Translate words into formulas.

In the following, an optimization problem is given. We would like to relate the phrases given in the problem statement to the steps of the optimization strategy.

- Find the dimensions of a $\boxed{\text{rectangle with area 36 ft}^2}$ whose perimeter is as small as possible.

(a) With which step of the strategy does the boxed piece match and why?

Constraint, b/c it's a given fixed value.

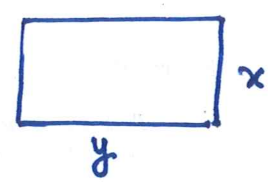
- Find the dimensions of a rectangle with area 36 ft^2 whose $\boxed{\text{perimeter is as small as possible}}$.

(b) With which step of the strategy does the boxed piece match and why?

the quantity to be optimized (minimized) $\rightsquigarrow$ objective function.

(c) *Objective function*: Write a formula that represents the quantity to be optimized.

$$\boxed{\underset{P}{\text{perimeter}} = 2x + 2y}$$



(d) *Constraint*: Write an auxiliary formula given by the constraint and use that to write the objective function in terms of one variable.

$$A = \boxed{xy = 36} \longrightarrow y = \frac{36}{x}$$

(e) Do the rest of the steps and solve the problem completely.

$$P = 2x + 2\left(\frac{36}{x}\right) = 2\left(x + \frac{36}{x}\right)$$

continue next class.