

Name (print):

Student number:



University of British Columbia

MATH 110: APRIL EXAM

Date: *April 24, 2014*

Time: *3:30 p.m. to 6:00 p.m.*

Number of pages: *13 (including cover page)*

Exam type: *Closed book*

Aids: *No calculators or other electronic aids*

Rules governing formal examinations:

Each candidate must be prepared to produce, upon request, a UBC card for identification.

No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.

Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:

- *Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;*
- *Speaking or communicating with other candidates;*
- *Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.*

Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

For examiners' use only		
Question	Mark	Possible marks
1		8
2		7
3		17
4		6
5		7
6		6
7		5
8		4
9		5
Total		65

You do not have to simplify your answers. “Calculator-ready” answers are sufficient.

1. Determine whether each of the following statements is true or false. If it is true, provide justification. If it is false, provide a counterexample.

(a) [**2 marks**] If f is defined on the closed interval $[-1, 1]$, then f attains a global maximum on that interval.

(b) [**2 marks**] If f is differentiable everywhere and $f(x) = 0$ has two solutions, then $f'(x) = 0$ has at least one solution.

(c) [**2 marks**] $f(x) = \sin x$ has infinitely many inflection points.

(d) [**2 marks**] If $f(1) = 2$, then f does not have a vertical asymptote at $x = 1$.

2. For each of the following short answer questions, underline your final answer.

(a) [**2 marks**] Find $\lim_{x \rightarrow 2} \frac{x^4 - 8x}{x^2 - 4}$.

(b) [**3 marks**] Find $\lim_{x \rightarrow \infty} \frac{5x^2 + \ln x}{2x^2 + 3x}$.

(c) [**2 marks**] State either the Mean Value Theorem or Rolle's Theorem.

3. Let $f(x) = \frac{x^2}{(x-4)^2}$.

(a) [**1 mark**] State the domain of f .

(b) [**3 marks**] Find all the vertical and horizontal asymptotes of f , if there are any.

(c) [**2 marks**] Compute $f'(x)$.

(d) [**3 marks**] Find the intervals where f is increasing and the intervals where it is decreasing.

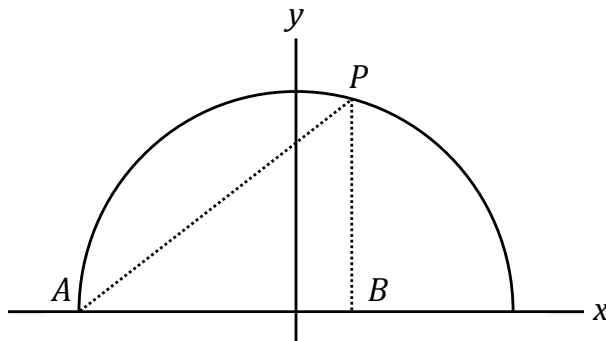
(e) [**2 marks**] Compute $f''(x)$.

(f) [**3 marks**] Find the intervals where f is concave up and the intervals where it is concave down.

(g) [**3 marks**] Make a large sketch of the graph of the function.

4. **[6 marks]** A farmer wants to enclose a rectangular pasture of 500 square metres. Three sides of the pasture will be enclosed with cedar fencing at a cost of \$10 per metre. The remaining side is to be enclosed with a stone wall at a cost of \$15 per metre. What should the dimensions of the field be to minimize the cost of the enclosure?

5. [7 marks] Find the coordinates of the point P on the semicircle $y = \sqrt{1 - x^2}$ of radius 1 (pictured below) for which the right triangle ABP has maximal area.



6. **[6 marks]** At noon, ship A is 30 kilometres west of ship B . Ship A is sailing east at 20 kilometres per hour and ship B is sailing north at 30 kilometres per hour. How fast is the distance between the ships changing at 4:00 p.m. on the same day? (You should indicate in your answer if the distance between the ships is increasing or decreasing.)

7. A particle moves across the x - y plane following the equation $10x = e^{2y} - e^{-2y}$.

(a) [**1 mark**] Verify algebraically that the particle crosses the point $(0, 0)$.

(b) [**4 marks**] Suppose we know that when the particle is at the point $(0, 0)$, the rate of change of its x -coordinate is 4 metres per second. What is the rate of change of its y -coordinate at that point?

8. Imagine bushbuck antelopes interacting in a montane forest. Let P denote the probability that a given antelope is in contact with another antelope. P is modelled by the equation

$$P = 1 - e^{-\pi\rho D^2},$$

where D is a constant denoting the “spotting distance” of the species, and ρ is a function denoting the density of antelope in the area.

- (a) [**2 marks**] Suppose $\rho'(t)$ is determined by field observations. Come up with an expression for the rate of change of the probability that an individual is in contact with another individual at time t . (Your answer will involve the term $\rho'(t)$.)

- (b) [**2 marks**] Now suppose $\rho'(t) = 0$ for a period of time from $t = a$ to $t = b$. Make a large sketch of the graph of P during that period.

9. (a) [**3 marks**] Use a linear approximation to estimate $\ln(0.9)$.

(b) [**1 mark**] Is your answer in part (a) an overestimate, an underestimate, or exactly equal to, the actual value of $\ln(0.9)$? Justify your answer.

(c) [**1 mark**] In some cases, the linear approximation of a function is equal to the function itself for all values of x . What is one such function?

This page may be used for rough work. It will not be marked.