# MATH 190, Homework 1

## Due date: Monday, Sept 17, 2018 (in class)

---

*Hand in full solutions to the questions below. Make sure you justify all your work and include complete arguments and explanations. Your asnwers must be clear and neatly written, as well as legible (no tiny drawings or micro-handwriting please!). Your answers must be* ***stapled****, with your name and student number at the top of each page.*

1. Find all $x \in \mathbb{R}$ that solves the following equation.

$$x^4 + 3x^2 - 10 = 0.$$

2. Recall that the absolute value function is defined by

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

   (a) Plot $|x|$

   (b) Plot the piecewise function

$$f(x) = \begin{cases} |x|, & x \geq -2 \\ 2x + 6, & -4 \leq x < -2 \\ -2 & x < -4 \end{cases}$$

   (c) Use the graph to find the domain and range of $f$.

   (d) Use the graph to find the zeros (roots) of $f$.

3. Consider the functions

$$f(x) = x^2 + 6x \quad \text{and} \quad g(x) = 3 - 2x$$

   Find all real numbers $x$ such that $f \circ g(x) = g \circ f(x)$.

4. Using the relevant graphs/triangles/unit circle explain why

$$\sin\left(\frac{11\pi}{6}\right) = -\frac{1}{2}.$$

5. Consider the following functions

$$g(x) = 3\cos(2018x)$$

and

$$f(x) = \begin{cases} x^4 + 8x^2, & x > 3 \\ 5 & -3 \le x \le 3 \\ 2^x & x < -3 \end{cases}$$

Determine the *range* of the function $f \circ g$. Ensure your answer is fully justified.

6. Below, the graph of functions $f$ and $g$ are given. Using the graph answer the following questions.

   (a) Write the domain and range of $f$. Dom: $[0,6]$ , Range: $[-1,1]$

   (b) Write the domain and range of $g$. Dom: $[0,6]$ , Range: $[-2,2]$

   (c) Evaluate

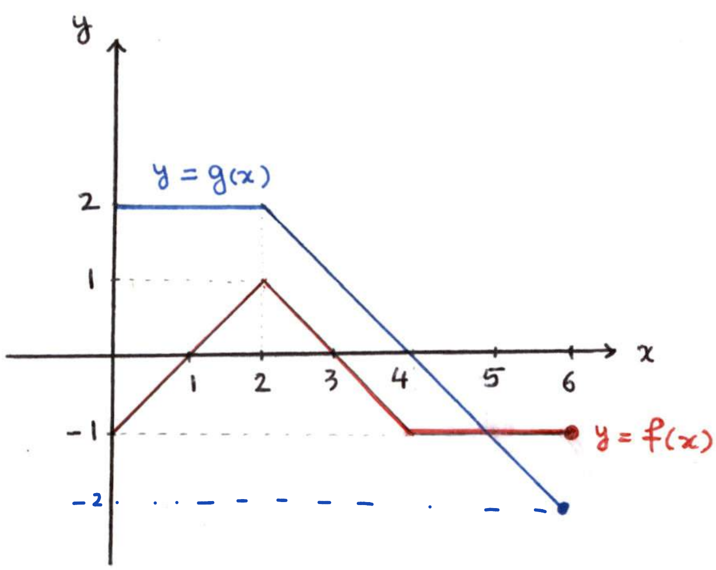
      i. $f \circ g(0) = f(g(0)) = f(2) = 1$

      ii. $f \circ g(4) = f(g(4)) = f(0) = -1$

      iii. $g \circ g(4) = g(g(4)) = g(0) = 2$

      iv. $g \circ f(4) = g(f(4)) = g(-1) =$ undefined $\rightarrow -1$ is NOT in the domain of $g$.

      v. $f \circ g \circ f(3) = f(g(f(3))) = f(g(0)) = f(2) = 1$



$$x^4 + 3x^2 - 10 = 0.$$

take $x^2 = t$, then the equation becomes

$t^2 + 3t - 10 = 0 \Rightarrow (t+5)(t-2) = 0$

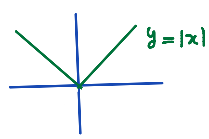
$\rightarrow t+5 = 0 \Rightarrow t = -5$

$\rightarrow t-2 = 0 \Rightarrow t = 2$

$\overset{t=x^2}{\Rightarrow}$ $x^2 = -5$ NO Solution $\rightsquigarrow$ squared number = negative number !!!

$\Rightarrow$ $x^2 = 2$ $\Rightarrow$ $\boxed{x = \sqrt{2}}$ or $\boxed{x = \sqrt{2}}$

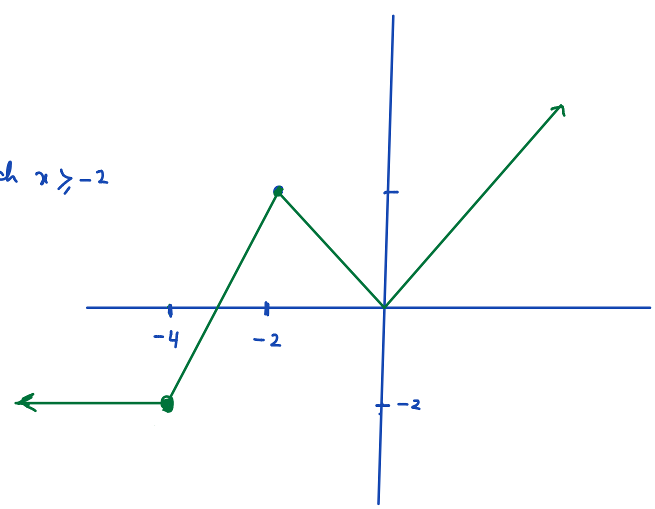
2) $f(x) = |x|$




only pick the part of $|x|$ for which $x \geq -2$

$$f(x) = \begin{cases} |x|, & x \geq -2 \\ 2x+6, & \underbrace{-4 \leq x < -2} \\ -2 & x < -4 \end{cases}$$

pick the piece of line with these x-values




(c) Domain : $(-\infty, \infty)$ or $\mathbb{R}$

Range : $[-2, \infty)$ or $y \geq -2$

(d) Roots are x-intercepts : line $y = 2x+6$ crosses x-axis when

$2x + 6 = 0 \Rightarrow 2x = -6 \Rightarrow x = \frac{-6}{2} = -3$

Also the graph has a root at the origin

$\Rightarrow$ roots : $\boxed{x = -3, \ x = 0}$

$f(x) = x^2 + 6x$ and $g(x) = 3 - 2x$

$$f \circ g(x) = f(g(x)) = f(3-2x) = (3-2x)^2 + 6(3-2x)$$

$$g \circ f(x) = g(f(x)) = g(x^2+6x) = 3 - 2(x^2+6x)$$

Equate and find $x$:

$$(3-2x)^2 + 6(3-2x) = 3 - 2(x^2+6x)$$

$$9 - 12x + 4x^2 + 18 - 12x = 3 - 2x^2 - 12x$$

$\xRightarrow[\text{re-order}]{\text{group}}$ $6x^2 - 12x + 24 = 0$

$\Rightarrow$ $6(x^2 - 2x + 4) = 0$

$\xRightarrow{\div 6}$ $x^2 - 2x + 4 = 0$

$\Rightarrow$ factoring does NOT work $\rightsquigarrow$ quadratic formula: $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

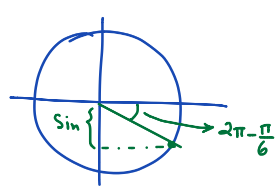
but $b^2 - 4ac = (-2)^2 - 4(1)(4) = -12 < 0$

$\Rightarrow$ NO solution

$\Rightarrow$ For NO $x$, $f \circ g = g \circ f$

(4) With the unit circle:

$$\frac{11\pi}{6} = \frac{12\pi - \pi}{6} = 2\pi - \frac{\pi}{6} \rightsquigarrow \frac{\pi}{6} \text{ less than one complete cycle}$$




$$\sin\left(\frac{11\pi}{6}\right) = \underset{\text{quadrant}}{-} \underbrace{\sin\frac{\pi}{6}}_{\text{reference}} = -\frac{1}{2}$$

$$g(x) = 3\cos(2018x)$$

$$f(x) = \begin{cases} x^4 + 8x^2, & x > 3 \\ 5 & -3 \le x \le 3 \\ 2^x & x < -3 \end{cases}$$

$$f \circ g(x) = f(g(x)) = f(\underbrace{3\cos(2018x)}_{\text{input of which } f\ ?})$$

We know for any angle $\theta$

$-1 \le \cos\theta \le 1$ so $-1 \le \cos(2018x) \le 1$

$\Rightarrow -3 \le 3\cos(2018x) \le 3$

$$\Rightarrow f(\underbrace{3\cos(2018x)}_{\substack{\text{This is a value between } -1 \text{ and } 1 \\ \text{So we input it in the } 2^{nd} \text{ line of } f}}) = 5$$

$\Rightarrow f \circ g(x) = 5$ for any $x$ $\Rightarrow$ Range of $f \circ g = \{5\}$