MATH 190, Homework 2

Due date: Monday, Oct 1, 2018 (in class)

Hand in full solutions to the questions below. Make sure you justify all your work and include complete arguments and explanations. Your asnwers must be clear and neatly written, as well as legible (no tiny drawings or micro-handwriting please!). Your answers must be ***stapled****, with your name and student number at the top of each page.*

1. Find all x satisfying each of the following equations.

   (a) $\tan x \sin x = \sqrt{2} \tan x$

   (b) $e^{2x} - 6e^x + 9 = 0$

   (c) $6x^{\frac{5}{3}} + 4x^{\frac{2}{3}} = 0$

   (d) $\ln(3x - 1) - \ln(3) = 2$

   (e) $\ln\left((x-1)^{x-1}\right) = 0$

   (f) $\log_3(\log_5 x) = 1$

2. Find all (real) zeros of the function

$$h(x) = \tan\left(\frac{1}{x}\right) \cdot \ln(x^2 + x - 1)$$

3. Many natural phenomena obey power rules. That is

$$Y = CX^m$$

   where C and m are constants which depend on the particular application. For example in physics we have the Stephan-Boltzmann equation where Y is the power emitted by a star with temperature X. In forestry we have models of tree size distribution where Y is the number of trees with stem size X. Other examples include frequency of words in most languages, population of cities, and rate of reaction in chemistry.

   (a) Let $y = \ln Y$ and $x = \ln X$. Express y in terms of x assuming that $Y = CX^m$. Note that C and m are fixed constants.

   (b) Suppose we made a plot of y as a function of x. Describe this plot. (shape, any intercept, etc.)

There is another question on the next page.

4. In this problem we will prove change of base formula for converting logarithms from one base to another. Show that for $a > 0, a \neq 1$ and $b > 0, b \neq 1$ we have

$$\log_b x = \frac{\log_a x}{\log_a b}.$$

Hint: First let $z = \log_b x$ and rewrite it using exponents instead of logarithm, then take logarithm in base a to achieve the identity. Make sure that you explain each step.

Bonus. Prove the logarithm identity for powers with a similar argument.

$$\log_b x^n = n \log_b x$$