Lecture 2
Sept 7

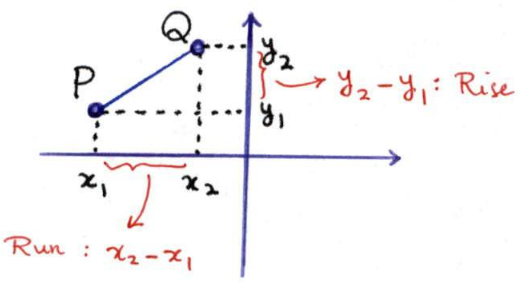
Reminder :

* Labs begin next week.
* HW1 will be posted on Mon, Sept 10.
  Due date : Sept 17
* Quiz 1 : Mon, Sept 24

Lines.

Slope of a line : A quantity that measures how fast the line is rising or falling moving from left to right.

Given two points $P=(x_1, y_1)$ and $Q=(x_2, y_2)$, the slope of the line segment PQ (secant line) is given by



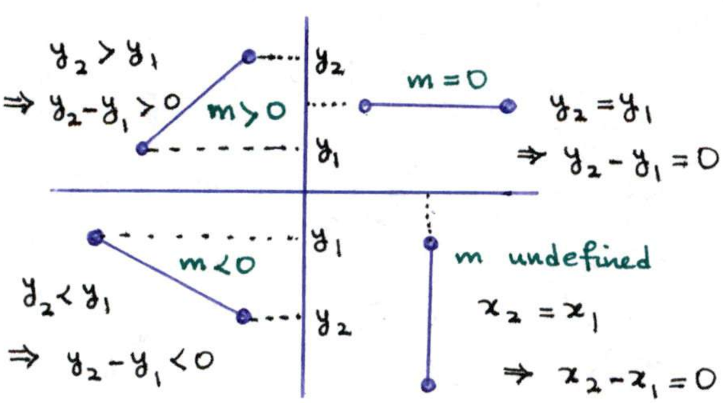
$$\text{slope of PQ} = \frac{\text{Rise}}{\text{Run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{\Delta y}{\Delta x}$$

denotes change in a quantity.

* One common notation for slope $= m$.

Sign of the slope :



* positive slope $\rightsquigarrow$ Rising line
* negative slope $\rightsquigarrow$ Falling line
* slope $= 0$ $\overset{\Delta y = 0}{\rightsquigarrow}$ Horizontal line
  numerator 0 is OK
* Slope undefined $\overset{\Delta x = 0}{\rightsquigarrow}$ Vertical line
  denominator 0 !!!

Equation of the line :

To find the equation of a line we need two pieces of info:

(1) slope of the line : m

(2) A point on the line : $P = (x_1, y_1)$

Then the equation of the line is :

$$\boxed{y - y_1 = m(x - x_1)}$$ → slope-point formula

Example 1 : Find the equation of a line with slope 3 that goes through the point $(2,5)$.

Solution:

$m = 3$

$P = (\underbrace{2}_{x_1}, \underbrace{5}_{y_1})$ $\xrightarrow[\text{we need}]{\text{we have all}}$ $y - y_1 = m(x - x_1)$

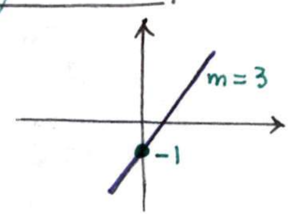
$\Rightarrow \quad y - 5 = 3(x - 2)$

Now let's simplify the equation, by distributing 3 and have y at one side and every other term at the other side of $=$.

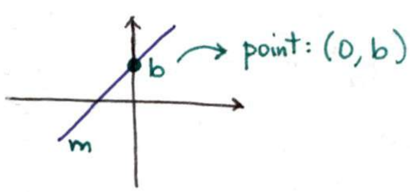
$$y - 5 = 3x - 6 \Rightarrow y = 3x - 6 + 5 \Rightarrow \boxed{y = \underbrace{3x}_{m} - 1}$$ → y-intercept

* The line equation in the form :

$$\boxed{y = mx + b}$$



is called the slope-intercept formula . This is because the value b is actually the y-intercept of the line i.e. the line crosses the y-axis at b at which $x = 0$



Example 2. What is the equation of the line through $(-1,1)$ and $(1,-5)$?

$\boxed{\text{Solution:}}$ We need to find the two pieces $\to m$, $\to P=(x_1,y_1)$

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{-5-1}{1-(-1)} = \frac{-6}{2} = -3$$

You can pick any of the points as (x_1,y_1) and (x_2,y_2) but be consistent.

$(1,-5) = (x_2,y_2)$

$(-1,1) = (x_1,y_1)$

$P = (1,-5) \Rightarrow y - y_1 = m(x-x_1)$

$\Rightarrow y-(-5) = -3(x-1)$

$\Rightarrow y+5 = -3x+3$

$\Rightarrow \boxed{y = -3x-2}$

* You can pick $(-1,1)$ and write the equation ⇝ The same answer

Example 3. What is the slope and y-intercept of the line

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$$2x - 3y = 6$$

A. $m=2$, y-int $=6$

B. $m=\frac{2}{3}$, y-int $=-2$

C. $m=\frac{3}{2}$, y-int $=-2$

D. $m=3$, y-int $=6$

E. None of the above.

Rewrite the equation in the slope-intercept form:

$$2x-3y=6 \Rightarrow -3y=-2x+6 \xRightarrow{\div -3} y = \frac{-2}{-3}x + \frac{6}{-3}$$

everything must be divided by 3.

$$\Rightarrow y = \frac{2}{3}x - 2$$

Compare with $y = mx + b$

$\boxed{B}$

Example 4 . Which of the following lines is parallel to

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$$y - 2 = \frac{5}{2}(x+1) \rightsquigarrow \text{What's } m\text{? Coefficient of } x$$

$\Rightarrow m_1 = \frac{5}{2}$

A. $y + 4 = \frac{2}{5}(x-7)$ $m_A = \frac{2}{5}$

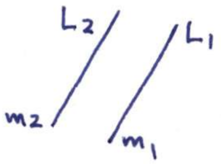
C. $y = \frac{5}{2}(x+7)$ $m_C = \frac{5}{2}$

B. $y + 1 = -\frac{5}{2}(x+1)$ $m_B = -\frac{5}{2}$

D. $y - 6 = -\frac{2}{5}(x-4)$ $m_D = -\frac{2}{5}$

Which one is perpendicular?

* Parallel lines have equal slopes



$L_1 \parallel L_2 \Rightarrow m_1 = m_2$

* $m_1 = m_C$

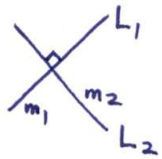
$\Rightarrow$ parallel to $\boxed{C}$

* $m_1 = -\frac{1}{m_D}$

$\Rightarrow$ perpendicular to $\boxed{D}$

* Perpendicular lines have slopes negative and reciprocal of each other.

$L_1 \perp L_2 \Rightarrow m_1 = -\frac{1}{m_2}$



Practice problem . Find the equation of the line that goes through the point $(1,2)$ and is perpendicular to the line $2x - 3y = 10$.

Solution: $P = (1,2)$

the other piece is m $\rightsquigarrow$ The line we look for is perp to $2x - 3y = 10$

so they must satisfy $m_1 = -\frac{1}{m_2}$ for their slope.

$$2x - 3y = 10 \xrightarrow{y = mx+b} 2x - 10 = 3y \Rightarrow \boxed{\frac{2}{3}}x - \frac{10}{3} = y$$

Slope of perp. line

so $\boxed{m = -\frac{3}{2}}$

and $\boxed{P = (1,2)}$

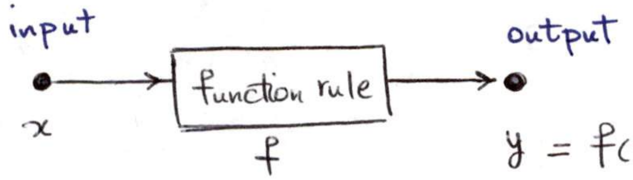
$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{3}{2}(x-1)$$

$$\Rightarrow \boxed{y = -\frac{3}{2}(x-1) + 2}$$

Functions

What is a Function? A function is a rule or relation that takes an input and assigns to it a unique output.

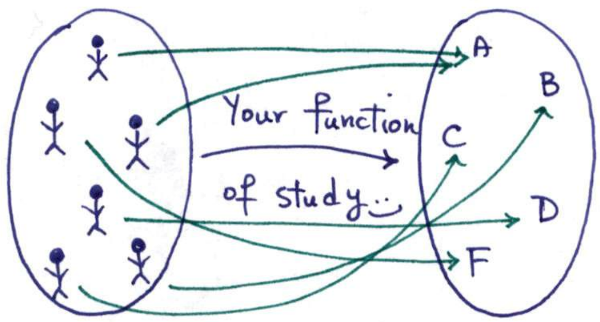


$y = f(x) \rightsquigarrow$ y is produced from acting function f on input x.

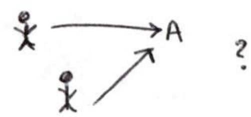
Examples of functions:

(1) Set of Inputs: Students in our course

Set of Outputs: Final grade

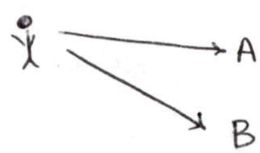


* Is it OK to have



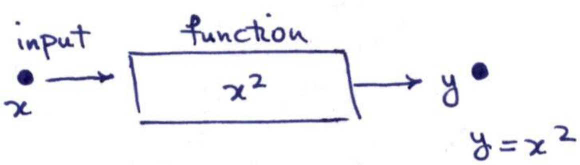
Yes, still each input one unique output.

* What about?



NO, one input with two outputs $\rightsquigarrow$ NOT a function

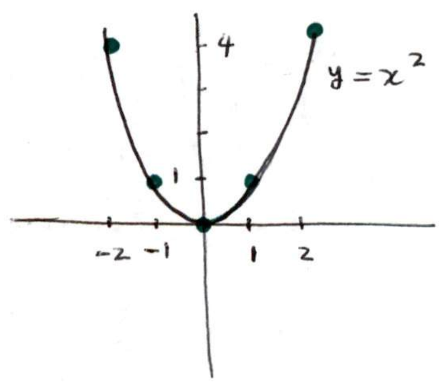
(2) $f(x) = x^2$



$1 \longrightarrow \boxed{f(1) = 1^2} \longrightarrow 1^2 = 1 \rightsquigarrow f(1) = 1$

$-2 \longrightarrow \boxed{f(-2) = (-2)^2} \longrightarrow (-2)^2 = 4 \rightsquigarrow f(-2) = 4$

$\frac{1}{3} \longrightarrow \boxed{f(\frac{1}{3}) = (\frac{1}{3})^2} \longrightarrow (\frac{1}{3})^2 = \frac{1}{9} \rightsquigarrow f(\frac{1}{3}) = \frac{1}{9}$



⊛ The points help us sketch the graph of the function $f(x) = x^2$

Question : Come up with a relation/rule that is NOT a function.

Notation for Domain is usually X.

Domain : The set of all <u>acceptable inputs</u> that a function can take.
and

Range : The set of all produced outputs.

usually denoted by Y.

Example 5 . How many of the following graphs are graph of a function? Each input must go to <u>only one</u> output.

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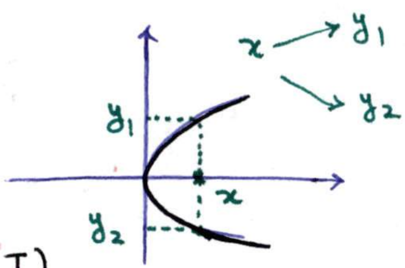
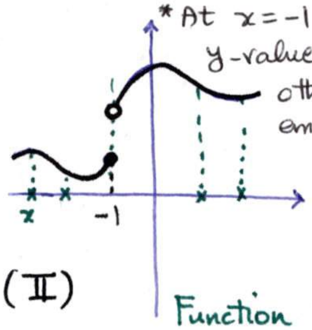
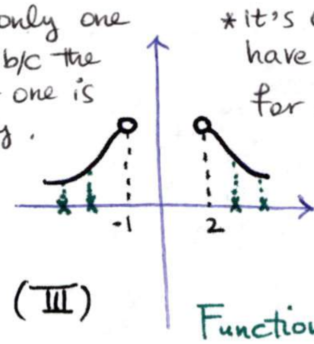


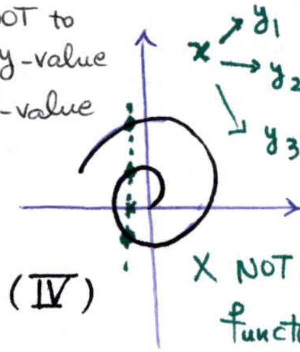
(I) X NOT a function



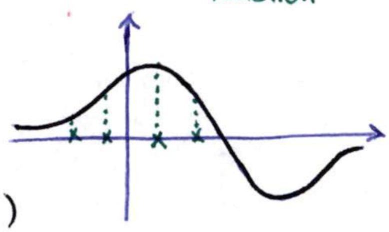
(II) Function



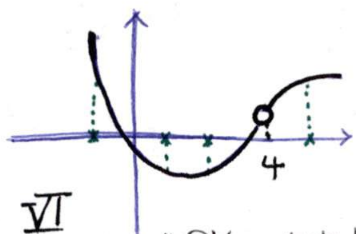
(III) Function



(IV) X NOT a function



(V) Function



VI Function

* OK not to have any y-value at $x=4$

A. 3

B. 2

C. 4

D. None

E. All

$\Rightarrow$ $\boxed{C}$

How to test a graph for a function?

Vertical Line Test :

A given graph is the graph of a function, if any vertical line intersects the graph at most at one point

* Discontinuities and jumps in the graph are OK, as long as we don't get more than y-value for any x-value.