Please hand in your homework

$\boxed{\text{Reminder}}$:

Quiz 1: Next Monday, Sept 24
topics: trigonometry and exponential functions.

Clicker Q: Which of the following is true?

(I) $\cos\left(\frac{15\pi}{2}\right) = 0$

(II) $\sin\left(\frac{15\pi}{2}\right) = 1$

(III) $\sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

(IV) $\sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

A. I and II

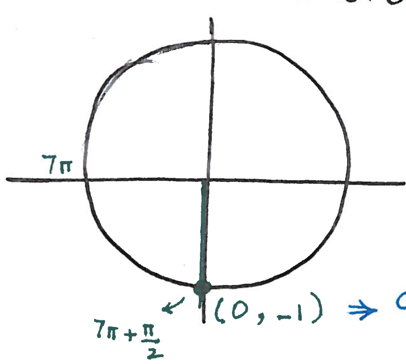
B. III and IV

C. II, III and IV

$\boxed{\text{D.}}$ I, III and IV

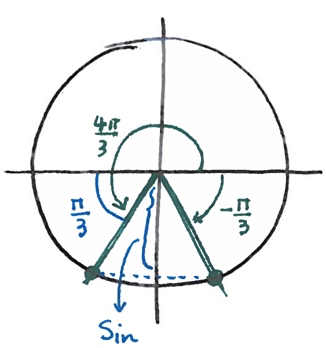
E. All of them

First we find the angles in the unit circle, then we find the endpoint corresponding to the terminal side of the angle and we read its x and y coordinates: $x = \cos\theta$, $y = \sin\theta$



$\theta = \frac{15\pi}{2} = \frac{14\pi + \pi}{2} = \frac{14\pi}{2} + \frac{\pi}{2} = \underbrace{7\pi}_{\text{7 half circles}} + \frac{\pi}{2} \rightsquigarrow \frac{3\pi}{2}$ reference

$(0,-1) \Rightarrow \cos\left(\frac{15\pi}{2}\right) = 0$, $\sin\left(\frac{15\pi}{2}\right) = -1 \Rightarrow$ II is wrong



$\theta = -\frac{\pi}{3} \rightsquigarrow \frac{\pi}{3}$ in $\ominus$ direction $\rightsquigarrow$ 4^{th} quadrant $\rightsquigarrow$ Sin: $\ominus$
reference

table: $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \rightsquigarrow \sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2} \rightsquigarrow$ III ✓

$\theta = \frac{4\pi}{3} = \frac{3\pi + \pi}{3} = \pi + \frac{\pi}{3} \rightsquigarrow 3^{rd}$ quad $\rightsquigarrow$ Sin: $\ominus$
reference

table: $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \Rightarrow \sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2} \rightsquigarrow$ IV ✓

Trigonometric Equations

Example 1. Find all real x values in $[0, 2\pi]$ such that

$$2\cos x \sin x + \sqrt{3}\cos x = 0$$

?What is the angle x that solves this equation:

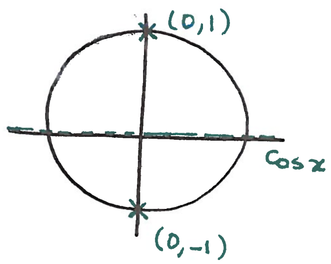
$$\cos x\,(2\sin x + \sqrt{3}) = 0$$

$\cos x = 0$ or $2\sin x + \sqrt{3} = 0$

Case $\cos x = 0$:

$\cos x = 0$

* Which angle has $\cos = 0$?



$\Rightarrow \quad x = \frac{\pi}{2}, \quad x = \frac{3\pi}{2}$

Case $2\sin x + \sqrt{3} = 0$:

$2\sin x + \sqrt{3} = 0$

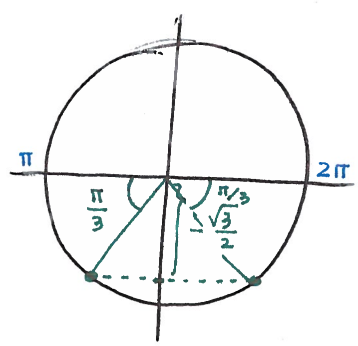
$\Rightarrow \quad 2\sin x = -\sqrt{3}$

$\Rightarrow \quad \sin x = -\frac{\sqrt{3}}{2}$

* Angles whose sin is $-\frac{\sqrt{3}}{2}$?

reference : $\frac{\pi}{3}$

3rd or 4th quadrant



Compute from the start axis ↝ positive axis

$$x = \pi + \frac{\pi}{3} = \frac{3\pi + \pi}{3} = \frac{4\pi}{3}$$

$$x = 2\pi - \frac{\pi}{3} = \frac{6\pi - \pi}{3} = \frac{5\pi}{3}$$

* Why don't we write it $-\frac{\pi}{3}$?

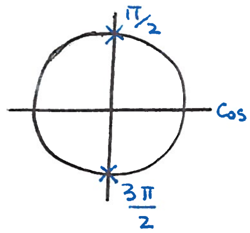
So the x-values that solve the equation are:

$$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{4\pi}{3}, \frac{5\pi}{3}}$$

Example 2. Find the domain of $y = \tan x$

We know $\tan x = \dfrac{\sin x}{\cos x}$, so we need to exclude the x-values that make the denominator $= 0$ from the domain, so let's solve

$\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$



⇝ These are only in one cycle, if we rotate on the unit circle and keep adding half-cycles to $\frac{\pi}{2}$ we'll get all the values in other cycle, we can do this in the negative direction as well so :

$\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{\pi}{2} \pm \pi, \frac{3\pi}{2} \pm \pi, \frac{5\pi}{2} \pm \pi, \ldots$

$\Rightarrow x = \frac{\pi}{2} + n\pi$ for $n = 0, \pm 1, \pm 2, \ldots$

$\Rightarrow$ $\boxed{\text{Domain} : \mathbb{R} - \left\{ \frac{\pi}{2} + n\pi, n = 0, \pm 1, \ldots \right\}}$

Example 3. Find all x in $\mathbb{R}$ satisfying

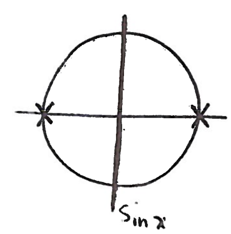
$$\cos x \sin x + 1 = 1$$

$$\cos x \cdot \sin x = 1 - 1 = 0$$

$\cos x = 0$ or $\sin x = 0$

Ex. 2 $\Rightarrow$ $x = \frac{\pi}{2} + n\pi$

$n = 0, \pm 1, \pm 2, \ldots$



$x = 0, \pi, 2\pi$

other cycles ⇝ $3\pi, 4\pi, \ldots$

Keep going and $\sin x = 0$ for $x =$ multiples of π

$\Rightarrow x = n\pi$, $n = 0, \pm 1, \pm 2, \ldots$

So $\boxed{x = n\pi \text{ or } x = \frac{\pi}{2} + n\pi \text{ for } n = 0, \pm 1, \pm 2, \ldots}$

Note. We can combine the two sets of angles into one :

$$x = \frac{n\pi}{2}$$

$n = 0, \pm 1, \pm 2, \ldots$