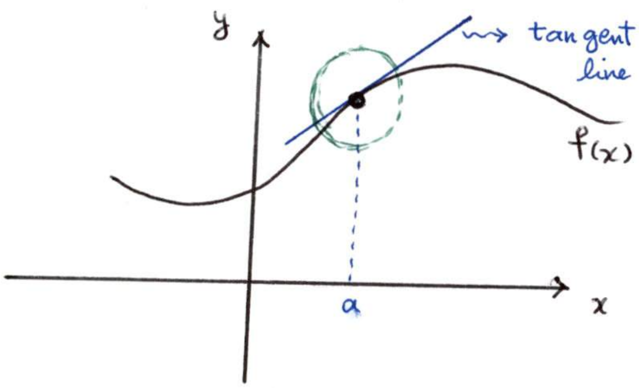
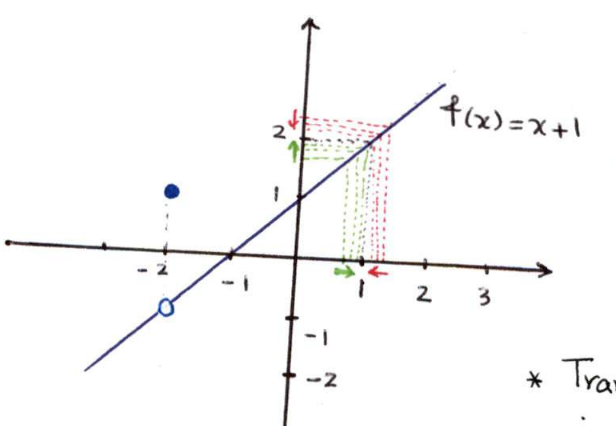
Recall our first class : Our main tool in differential calculus is derivative and derivative is based upon tangent line to the graph of a function $y = f(x)$.



Goal : info about the slope of the tangent line.

We need to have info about $f(x)$ "close" to the tangency point $x = a$ and for this we need $\boxed{\text{limits}}$.

To see what limit means, let's start with graph of a function :



Notation: $x \to 1$

Question 1: When $\boxed{x \text{ "gets close" to } 1}$

$f(x) \to 2$ $\boxed{f(x) \text{ " gets close" to } 2}$

* track function values on the y-axis when x gets close to 1.

* Translation of " As $x \to 1$ then $f(x) \to 2$ " in math language is :

$$\lim_{x \to 1} f(x) = 2$$

We can compute some values of $f(x)$ for x's close to 1 and see what's going on.

Let's make a table of values with some x close to 1 and find $f(x) = x + 1$.

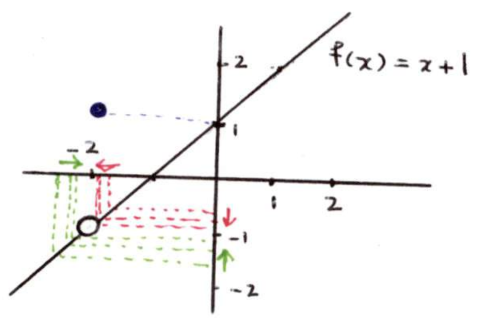
	x	$f(x) = x+1$	
approaching 1 from left	0.9	1.9	$f(x) \to 2$
	0.99	1.99	
approaching 1 from right	1.1	2.1	
	1.01	2.01	

$$\lim_{x \to 1} f(x) = 2.$$

Question 2.

When x "gets close" to -2, $f(x)$ "gets close" to ...-1

Again let's track y-values.



	x	$f(x) = x+1$	
from left of -2	-2.1	-1.1	$f(x) \to -1$
	-2.01	-1.01	
from right of -2	-1.9	-0.9	
	-1.99	-0.99	

But $f(-2) = 1$

* From graph and from the table of values it can be seen that:

As we approach -2 on the x-axis, the values of function on the y-axis approaches to -1 i.e. $\lim_{x \to -2} f(x) = -1$

Note that the exact value of the function and $x = -2$ is $f(-2) = 1$

So

* In finding limit the exact value at the given x-value does NOT matter. We are checking the values close to that x NOT exactly at x.

Limit

The limit is an operation that we perform on a function $y = f(x)$.

output variable

$$\lim_{x \to a} f(x) = L$$

a constant number on the y-axis.

input variable

a constant number on the x-axis

Meaning : As the x-values are getting closer and closer to the value "a", the y-values of the function are getting closer and closer to the value L on the y-axis.

⊛ Again, note that limit is all about being close to some value NOT exactly at that value.

Note : Different letters can be used for variable on the x-axis and functions and the given numbers.

* $\lim_{t \to a} g(t) = L$

input variable on the x-axis

* $\lim_{r \to c} h(r) = M$

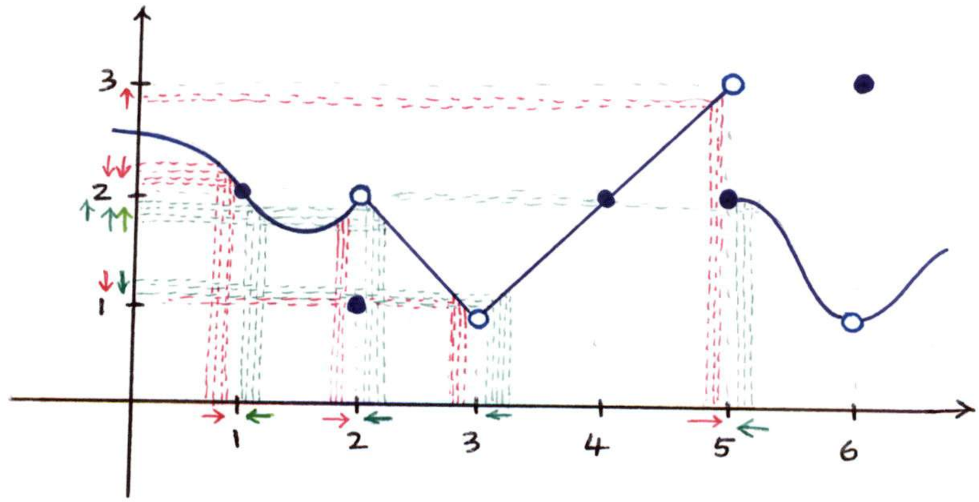
Some number on the x-axis

some number on the y-axis

⊛ Don't get confused with the notation.

⊛ Understand the concept and translate that to any mathematical notations.

Example : Let $f(x)$ be a function given by the following graph.



Evaluate the following limits :

(1) $\lim_{x \to 1} f(x) = 2$ $\quad$ * In this case : $\lim_{x \to 1} f(x) = 2 = f(1)$

A. 1 $\quad$ $\boxed{\text{B. 2}}$ $\quad$ C. 2.5 $\quad$ D. 3 $\quad$ E. Does NOT exist.

(2) $\lim_{x \to 2} f(x) = 2$ $\quad$ * In this case : $\lim_{x \to 2} f(x) = 2$ but $f(2) = 1$

A. 1 $\quad$ $\boxed{\text{B. 2}}$ $\quad$ C. 2.5 $\quad$ D. 3 $\quad$ E. Does NOT Exist.

(3) $\lim_{x \to 3} f(x) = 1$ $\quad$ * In this case : $\lim_{x \to 3} f(x) = 1$ but $f(3)$ is undefined.

$\boxed{\text{A. 1}}$ $\quad$ B. 2 $\quad$ C. 2.5 $\quad$ D. 3 $\quad$ E. Does NOT Exist (DNE)

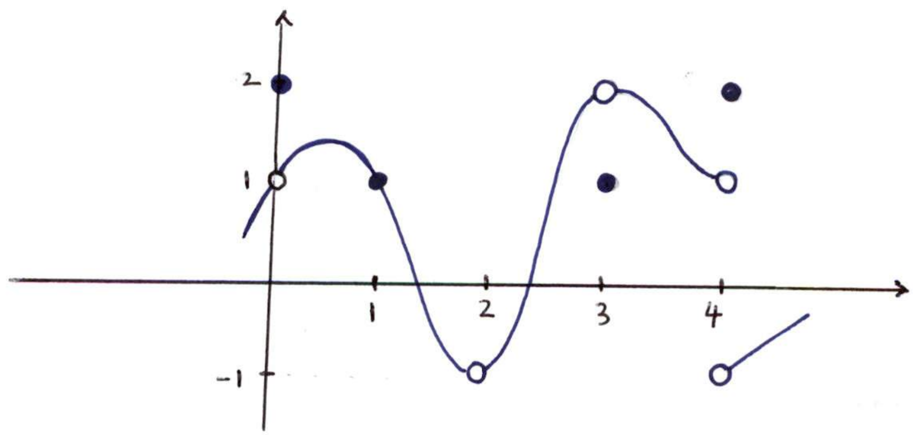
(4) $\lim_{x \to 5} f(x)$ = We get two different limits from left and right
$\rightsquigarrow$ NO unique limit

right limit $\quad \lim_{x \to 5^+} f(x) = 2$

left limit $\quad \lim_{x \to 5^-} f(x) = 3$

A. 1 $\quad$ B. 2 $\quad$ C. 2.5 $\quad$ D. 3 $\quad$ $\boxed{\text{E. DNE}}$

Practice : Use the following graph for the function $y = g(x)$ to find the limits.



- $\lim_{x \to 0} f(x) =$

  Compare with $f(0)$

- $\lim_{x \to 1} f(x) =$

  Compare with $f(1)$

- $\lim_{x \to 2} f(x) =$

  with $f(2)$

- $\lim_{x \to 3} f(x) =$

  with $f(3)$

- $\lim_{x \to 4} f(x) =$

  with $f(4)$