Solution (HW 2)

(1) (a) $\tan x(\sin x - \sqrt{2}) = 0$

$\tan x = 0$

$\Downarrow$

$x = 0, \pm\pi, \pm 2\pi, \ldots$

$\Rightarrow x = n\pi$ for $n = 0, \pm 1, \pm 2, \ldots$

$\sin x = \sqrt{2}$

$\Downarrow$

No solution

$\sqrt{2}$ is out of range of $\sin x$

(b) $e^{2x} - 6e^{x} + 9 = 0$

$e^x = t$

$\Rightarrow (e^x - 3)(e^x - 3) = 0 \Rightarrow e^x = 3 \Rightarrow x = \ln 3$

(c) $2x^{2/3}(3x + 2) = 0$

$\Rightarrow x^{2/3} = 0 \Rightarrow x = 0$

or $3x + 2 = 0 \Rightarrow x = \frac{-2}{3}$

(d) $\ln\left(\frac{3x-1}{3}\right) = 2 \xrightarrow[\text{to both sides}]{\text{raise } e} e^{\ln\left(\frac{3x-1}{3}\right)} = e^{2}$

$\Rightarrow \frac{3x-1}{3} = e^2 \Rightarrow 3x - 1 = 3e^2$

$\Rightarrow x = \frac{3e^2 + 1}{3}$

(e) Use ln rule : $\ln x^n = n \ln x$

$$(x-1)\ln(x-1) = 0 \quad \text{or} \quad \begin{cases} x-1=0 \Rightarrow x = 1 \rightsquigarrow \text{NOT acceptable} \\ \qquad \ln(1-1) = \ln 0 \ \times \\ \ln(x-1) = 0 \end{cases}$$

$$\xrightarrow{\text{raise } e} e^{\ln(x-1)} = e^0 \Rightarrow x-1 = 1$$

$$\Rightarrow x = 2$$

(f) $\log_3(\log_5 x) = 1$

Use the log property : $b^{\log_b x} = x$

$$\xrightarrow[\text{3 to both sides}]{\text{first raise}} 3^{\log_3(\log_5 x)} = 3^1 \Rightarrow \log_5 x = 3$$

$$\xrightarrow[\text{to both side}]{\text{Now raise 5}} 5^{\log_5 x} = 5^3$$

$$\Rightarrow x = 125$$

(2) $h(x) = 0$ or

$$\ln(x^2+x-1) = 0 \xrightarrow{\text{raise } e} x^2+x-1 = e^0 = 1$$

$$\Rightarrow x^2+x-2 = 0$$

$$\Rightarrow (x+2)(x-1) = 0$$

$$\Rightarrow \underbrace{x = -2, \quad x = 1}_{\text{Both OK}}$$

$$\tan\left(\frac{1}{x}\right) = 0$$

if $\frac{1}{x} = \theta$

$\Rightarrow \tan(\theta) = 0 \Rightarrow \theta = 0, \pm\pi, \pm 2\pi, \ldots$

So $\frac{1}{x} = 0, \pm\pi, \pm 2\pi, \ldots$

0 is NOT possible : $x = \pm\frac{1}{\pi}, \pm\frac{1}{2\pi}, \ldots$

or $x = \pm\frac{1}{n\pi}$

for $n = \pm 1, \pm 2, \ldots$
$(n \neq 0)$

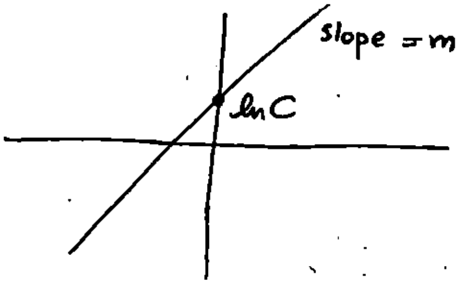
1st method: $Y = CX^m \xrightarrow[x = \ln X]{y = \ln Y}$ take ln $\ln Y = \ln(CX^m)$

$$\Rightarrow \quad y = \ln C + \ln X^m$$

$$= \ln C + m \ln X$$

$$= \ln C + mx$$

$\Rightarrow \quad y = mx + \ln C \rightsquigarrow$ This is an equation of a line.



y-intercept : $\ln C$

x-intercept : $-\dfrac{\ln C}{m}$

2nd method: $\begin{array}{l} Y = e^y \\ X = e^x \end{array} \overset{Y = CX^m}{\Rightarrow} \quad e^y = C(e^x)^m = Ce^{mx}$

Now take ln

$$\ln e^y = \ln Ce^{mx}$$

$$\Rightarrow \quad y = \ln C + \ln e^{mx}$$

$\Rightarrow \quad y = \ln C + mx$ Same equation.

(4) take $y = \log_b x$ then $x = b^y$ we want to get a log term in base "a", so take the $\log_a$ from both sides of

$$x = b^y$$

$$\log_a x = \log_a b^y = y \log_a b = \log_b x \cdot \log_a b$$

Solve for $\log_b x$:

$$\log_b x = \frac{\log_a x}{\log_a b} \checkmark$$

<u>Bonus</u>: take $y = \log_b x$ and translate it into exp

$$b^y = x$$

raise both sides to n: $(b^y)^n = x^n \Rightarrow b^{ny} = x^n$

translate back to $\log_b$: $\log_b x^n = ny$

but $y = \log_b x$ so $\log_b x^n = n \log_b x$ $\checkmark$