MATH 190, Homework 3

Due date: Monday, Oct 15, 2018 (in class)

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*Hand in full solutions to the questions below. Make sure you justify all your work and include complete arguments and explanations. Your asnwers must be clear and neatly written, as well as legible (no tiny drawings or micro-handwriting please!). Your answers must be **stapled**, with your name and student number at the top of each page.*

1. Compute the following limits.

   (a) $\lim_{x\to 1} \dfrac{\sqrt{x^2+2}-\sqrt{x+2}}{x-1}$

   (b) $\lim_{x\to 3} \dfrac{x^2-9}{|x-3|}$

2. Let $f(x) = \tan(x)$

   (a) Find all vertical asymptotes of $f(x)$.

   (b) Sketch the graph of $f(x)$.

   Hint: This function has infinitely many vertical asymptotes. Your solution must show the verification of one-sided limits for at least two of the vertical asymptotes to determine the sign of infinity.

3. Consider the function

$$g(x) = e^{-2x}\sin(2x)$$

   (a) What is domain of $g(x)$?

   (b) Does g have any vertical asymptotes? Why or why not?

   (c) Explain what happens to e^{-2x} as x approaches to ∞.

   (d) Explain what happens to $\sin(2x)$ as x approaches to ∞.

   (e) Using your answers from (c) and (d), explain what happens to $g(x)$ as x approaches to ∞.
   In this way you can suggest a value for horizontal asymptote of $g(x)$, what is this value?

4. Let $f(x) = \dfrac{3x+1}{x-1}$.

   (a) Use the limit definition of the derivative (and not any other method) to compute $f'(x)$.

   (b) Find the points at which the tangent line is horizontal.

   (c) Determine whether the graph of $f(x)$ is increasing or decreasing. Explain why.

5. For parts (a) and (b) below, sketch a graph of a function that satisfies **all** the given conditions:

   (a)
   - Domain of $f(x)$ is $\mathbb{R}$-$\{1\}$
   - $\lim_{x \to -2^+} f(x) = 2$ and $\lim_{x \to -2^-} f(x) = -1$.
   - $\lim_{x \to 1} f(x) = \infty$.
   - $\lim_{x \to 4} f(x) = 3$.
   - $f(-2) = -1$, $f(4) = 1$.

   (b)
   - Domain of $f(x)$ is $[-2, \infty)$.
   - f has a (two sided) vertical asymptote at $x = 1$.
   - f has a horizontal asymptote at $y = 2$.
   - $f(0) < 0$ and $f(3) > 0$.
   - $f'(0) = 0$ and $f'(3) = 0$.
   - $f'(x) < 0$ for all $-2 < x < 0$, and $f'(x) > 0$ for all $0 < x < 1$.
   - $f'(x) > 0$ for all $1 < x < 3$, and $f'(x) < 0$ for all $x > 3$.