Oct 24
Lecture 21

Midterm:

Next Monday, Oct 29, in class

Topics: Week 1 up to the end of Monday lecture

↳ Check this week's lab for a summary of topics covered in the midterm.

↳ Also check learning objectives for a list of goals you should be able to achieve up to the midterm.

Office Hours this week:

Thursday 11 am – 1 pm in MATX 1118

Friday 2:30 – 4 pm in LSK 300

Email me for an appointment

What to study:

- All lecture notes and examples therein.
- All lab questions
- Quizzes
- Homework problems
- posted practice problems & worksheets
- Sample midterm.

(*) You should re-do all the solved example in class, labs, HW, Quiz ...

(*) Reading the solutions is NOT sufficient and NOT effective at all.

(*) You should be able to solve the examples on your own with no solution checking.

(*) Always double-check your work, we all make algebra mistakes :(

Our last differential Calculus Topic is

Related Rates

Recall that instantaneous velocity is the slope of the tangent line which is the derivative of the distance.

Example we did: distance $x(t) = -10t^2 + 40t + 100$

V_{inst} at $t=1$ is $x'(1) = \frac{dx}{dt}$ at $t=1$

We use power rule to find $x'(t)$:

$$x'(t) = -20t + 40$$

plug $t=1$ $\Rightarrow$ $x'(1) = -20 + 40 = 20$ so $V_{inst} = 20$ m/s

⊛ For any physical quantity, the derivative gives the rate of change in that quantity:

V (velocity) = rate of change in distance $= \frac{dx}{dt} = x'(t)$

a (acceleration) = rate of change in velocity $= \frac{dV}{dt} = V'(t)$

area A ⇝ $\frac{dA}{dt}$ ⇝ rate of change in the area

Volume V ⇝ $\frac{dV}{dt}$ ⇝ rate of change in the volume

⊛ We'd like to use derivative to find the rate of change in some physical quantity.

Scenario: → A (physical) situation is given and it describes the change is some quantities over time. For example, change in distance, length, area, angle, radius, ... Some information about the value of quantities at one instant in time is also given.

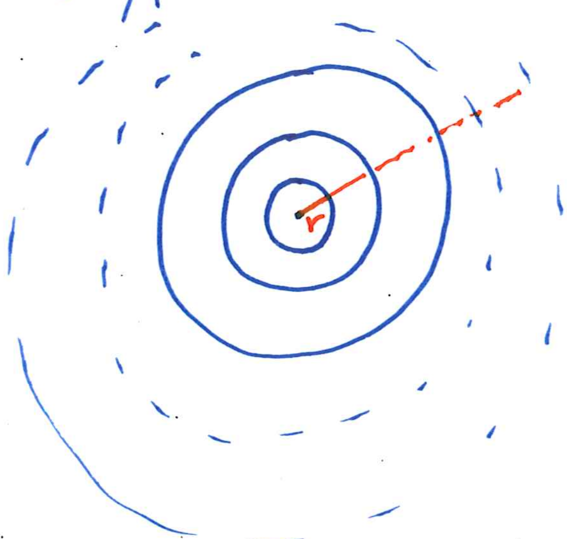
Question : If two or more quantities are related to each other by some mathematical formula, then find the rate of change (increase or decrease) in one of them assuming we know the rate of change of the others.

Example 1 : You walk alongside a calm lake and you throw a rock into the lake, so ripples in the shape of concentric circles are formed on the water.

having the same center

If the radius of a ripple is increasing at a rate of 3 inches per second, find the rate of increase in the area of the ripple when the radius is 6 inches.

1. Diagram/picture and notation.



- r = radius : changing with time
  $= r(t)$ increasing

- A = area of the circle
  $= A(t)$: increasing over time

2. Write an equation that relates the changing quantities.

$$A = \pi r^2$$

$$A(t) = \pi \underbrace{\left(r(t)\right)^2}$$

two functions composed : $t \xrightarrow{r(t)} r(t) \xrightarrow{t^2 = \circ^2} \left(r(t)\right)^2$

3. Given/Unknown values:

rate of increase in radius $= \dfrac{dr}{dt} = 3$ inch/sec

unknown: $\dfrac{dA}{dt}$ when $r = 6$ inch/sec

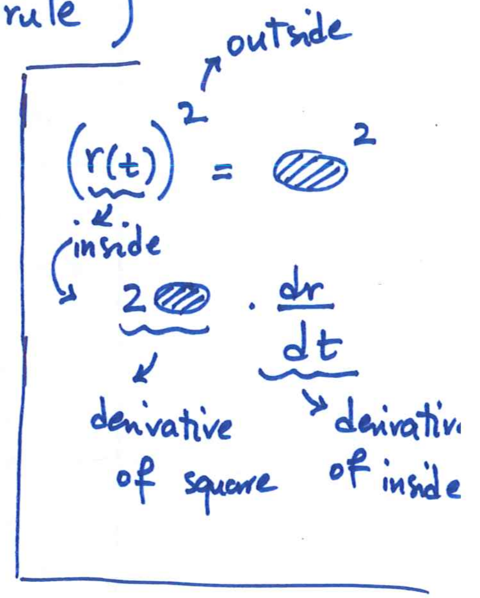
4. Differentiate the formula (Chain rule)

$$A(t) = \pi\,(r(t))^2$$

derive

$$\frac{dA}{dt} = \pi \cdot 2\,r(t) \cdot \frac{dr}{dt}$$

?   6   3



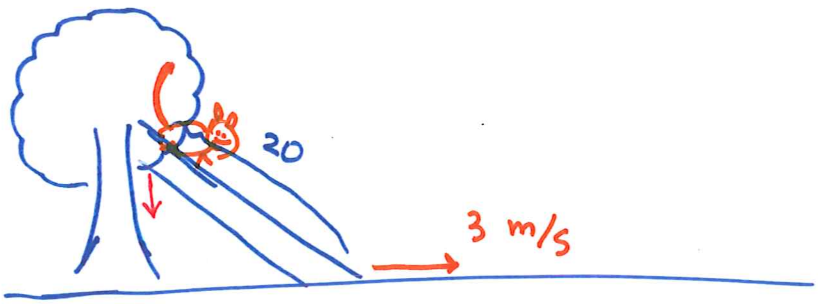
$$\frac{dA}{dt} = \pi \cdot 2 \cdot 6 \cdot 3$$

$$\frac{dA}{dt} = 36\pi \ \text{inch}^2/\text{sec}$$

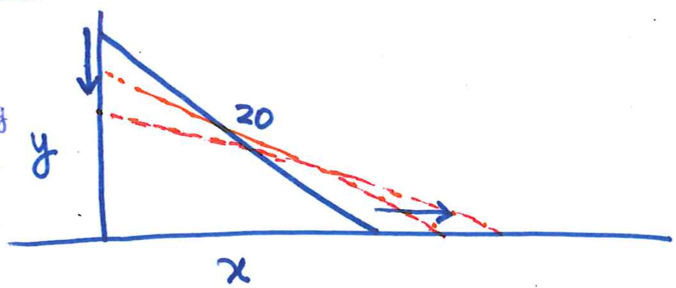
(*) We don't plug in given values $\underbrace{\text{before differentiation}}_{\text{for changing quantities}}$.

1st derive then substitute.

Example 2 : A 20 m - high fallen tree is resting against another tree. A squirrel is on top of the fallen tree whose base slides at a rate of 3 m/s. How fast is the squirrel descending when the base of the fallen tree is 12 m from the other.



- **Diagram** :



y is decreasing $\downarrow$ we expect $\frac{dy}{dt} < 0$

x is increasing so $\frac{dx}{dt} > 0$

- **Given info / unknown info** :

$$\begin{cases} \dfrac{dx}{dt} = 3 \text{ m/s} \\[2ex] \dfrac{dy}{dt} = \,? \quad \text{when } x = 12 \end{cases}$$

- **Relate the variables** : Pythagorean formula

$$x^2 + y^2 = 20^2 \longrightarrow \text{why did I sub 20 before differentiation?}$$

$$(x(t))^2 + (y(t))^2 = 20^2$$

- **Differentiate with the chain rule** :

$$2\underbrace{x}_{12}\overbrace{\frac{dx}{dt}}^{3} + 2\boxed{y}\cdot\overbrace{\frac{dy}{dt}}^{?\ \text{unknown}} = 0$$

missing info : we should find this first.

$\parallel$

16

$(x(t))^2 = ⬭^2$

derive $\hookrightarrow$ $2 ⬭ \cdot ⬭'$

$= 2x(t)\cdot x'(t)$

$= 2x(t)\cdot \frac{dx}{dt}$

Let's find y:

$$x^2 + y^2 = 20^2$$

$$\xrightarrow{x=12} \quad 12^2 + y^2 = 20^2 \Rightarrow y = \sqrt{400 - 144} = 16$$

Go back to the derivative:

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$2 \cdot 12 \cdot 3 + 2 \cdot 16 \cdot \frac{dy}{dt} = 0$$

solve for $\frac{dy}{dt}$

$$32 \frac{dy}{dt} = -72$$

$$\frac{dy}{dt} = \frac{-72}{32} = -\frac{9}{4} \text{ m/s}$$

decreasing distance.

* Be careful about decreasing quantities, their rate of change in negative.