MATH 190, Lab 6: Oct 16 and 18, 2018

Work through the following problems while the TAs circulate. When you have completed the problems (to the satisfactory of the facilitators) you can spend the rest of the lab working on the practice problems on the next page corresponding to the learning goals for the topic of limits and asymptotes.

Warm-up.

Recall the definition of the derivative of $y = f(x)$:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

Explain (using a picture) why this expression should give you the slope of the tangent line to $f(x)$ at the point $(x, f(x))$.

Problems.

1. Let $f(x) = \sqrt{x+3}$

   (a) Use the definition of the derivative above (not any other method) to find the derivative of $f(x)$.

   (b) Examining the derivative found in part (a), can you say whether $f(x)$ is always increasing or always decreasing?

   (c) What is the slope of the tangent line at $x = 6$.

   (d) Find the equation of the tangent line to $f(x)$ at $x = 6$.

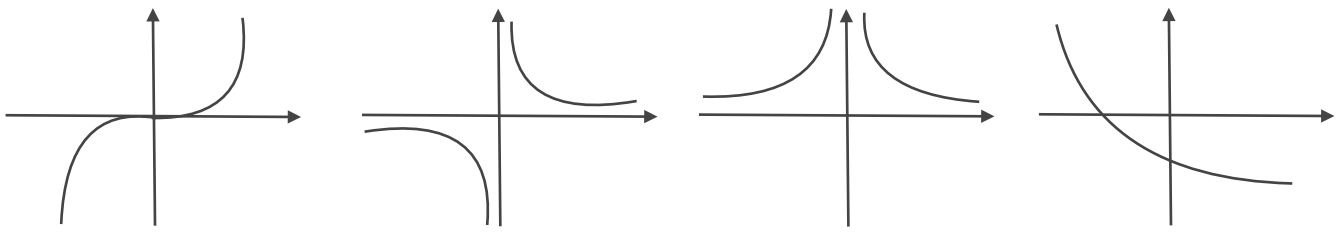
2. Let $f(x) = \dfrac{1}{x^3}$

   (a) Use *power rule* to find the derivative of $f(x)$.

   (b) What is the slope of the tangent line at $x = -2$.

   (c) Find the equation of the tangent line to $f(x)$ at $x = -2$.

   (d) Examining the derivative found in part (a), which of the following graphs can be the graph of $f(x)$?



A B C D

Learning Goals

Below, you can find a list of learning goals for the topic of limits that should be achieved. To make sure you understood the topic solve the practice problems corresponding to the goals.

Week 4 and 5: Limits and Asymptotes

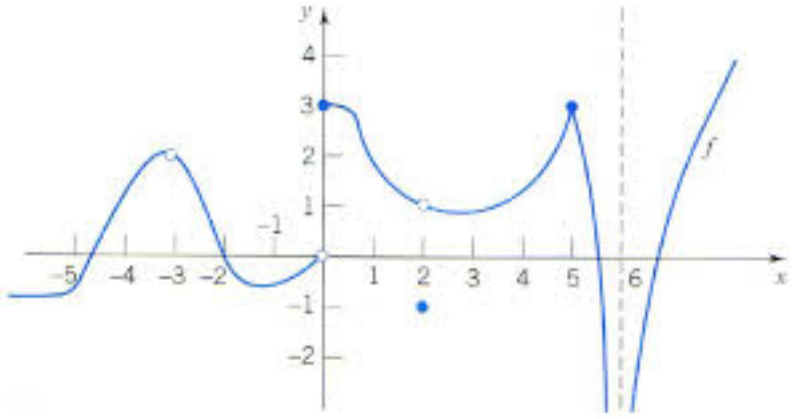
1. Explain using a picture what $\lim_{x\to a} f(x) = L$ means.
2. Explain using a picture what $\lim_{x\to a^-} f(x) = L$ and $\lim_{x\to a^+} f(x) = L$ means.
3. Evaluate limits using the graph of a function.

Sample Questions:

(a) TRUE/FALSE?

- $\lim_{x\to 4} f(x) = 1$ implies that $f(4) = 1$
- If $g(a) = 0$ then $\lim_{x\to a} \frac{f(x)}{g(x)}$ does NOT exist.
- If $\lim_{x\to 4} f(x) = 1$ then we must have $\lim_{x\to 4^+} f(x) = \lim_{x\to 4^-} f(x) = 1$
- If $g(x) = \begin{cases} \dfrac{x^2-6x+8}{x-4} & x < 4 \\ \sqrt{x} & x \geq 4 \end{cases}$, then $\lim_{x\to 4} g(x)$ does NOT exist.

(b) Use the graph to find the following limits.



- $\lim_{x\to -3} f(x) =$ $\quad f(-3) =$
- $\lim_{x\to 0^+} f(x) =$ $\quad \lim_{x\to 0^-} f(x) =$ $\quad \lim_{x\to 0} f(x) =$ $\quad f(0) =$
- $\lim_{x\to 2^+} f(x) =$ $\quad \lim_{x\to 2^-} f(x) =$ $\quad \lim_{x\to 2} f(x) =$ $\quad f(2) =$
- $\lim_{x\to 6^+} f(x) =$ $\quad \lim_{x\to 6^-} f(x) =$ $\quad f(6) =$

- $\lim_{x\to 5} f(x) =$ $\qquad$ $f(5) =$
- $\lim_{x\to\infty} f(x) =$ $\qquad$ $\lim_{x\to-\infty} f(x) =$

4. Evaluate limits of rational, trigonometric, exponential and logarithmic functions using algebraic techniques.

Sample Questions

(a) $\lim_{x\to 0} \dfrac{x^2-25}{x^2-4x-5} =$

(b) $\lim_{x\to 5} \dfrac{x^2-25}{x^2-4x-5} =$

(c) $\lim_{x\to -3} |x+1| + \dfrac{3}{x} =$

(d) $\lim_{x\to 2^+} \dfrac{-2x}{x^2-4} =$

(e) $\lim_{x\to 3} \dfrac{\sqrt{x^2+7}-3}{x+3} =$

(f) $\lim_{x\to 3} \dfrac{\sqrt{x+1}-2}{x^2-9} =$

(g) $\lim_{x\to 0} \sqrt{2\cos x + 5}$

(h) $\lim_{x\to 0} \dfrac{2^x+\sin x}{x^2-9} =$

(i) $\lim_{x\to 1^-} \dfrac{1}{x-1} + e^x$

(j) $\lim_{x\to 0} \dfrac{2^x+\sin x}{x^2} =$

(k) $\lim_{x\to 7} \sqrt[6]{2x-14}$

(l) $\lim_{x\to\infty} 2x^2 - 3x =$

(m) $\lim_{x\to\infty} \dfrac{x^4-10}{4x^3+x} =$

(n) $\lim_{x\to\infty} \dfrac{3x^3+x^2-2}{x^3+x-5} =$

(o) $\lim_{x\to\infty} \dfrac{x+5}{4x^2+1} =$

(p) $\lim_{x\to\infty} \dfrac{x-x^4}{6-x^2+5x^4} =$

5. Explain (either with a picture or with limits) the definition of vertical and horizontal asymptotes.

6. Given a function (rational or simple trig/exp/log) find the equations of any vertical and horizontal asymptotes.

Sample Questions

- Find the vertical asymptotes (if any) of functions in (a), (c), (d), (i), (j), (m), (o) above.
- Find the horizontal asymptotes (if any) of functions in (a), (d), (l), (m), (n), (o), (p) above.

Answer Key :

Goals 1-3 :

(a) F , F , T , F

(b)
- 2 , undefined
- 3 , 0 , DNE , 3
- 1 , 1 , 1 , -1
- $-\infty$, $-\infty$, undefined
- 3 , 3
- ∞ , -1

Goal 4 :

(a) 5
(b) $5/3$
(c) 1
(d) $-\infty$
(e) $\frac{1}{6}$
(f) $\frac{1}{24}$
(g) $\sqrt{7}$
(h) $-\frac{1}{9}$
(i) $-\infty$
(j) ∞
(k) 0
(l) ∞
(m) ∞
(n) 3
(o) 0
(p) $-\frac{1}{5}$

Goals 5 and 6 :

- (a) $x = -1$, (c) $x = 0$, (d) $x = 2$, $x = -2$, (i) $x = 1$, (j) $x = 0$
  (m) $x = 0$, (o) NO V.A.

- (a) $y = 1$, (d) $y = 0$, (l) NO H.A. , (m) NO H.A.
  (n) $y = 3$, (o) $y = 0$, (p) $y = \frac{-1}{5}$