

## Math 190: Final Exam Practice

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In addition to: Examples done in class, Lab problems, Homework problems, Quiz problems, Practice problems, consider solving the following problems.

1. Recall the derivatives of:  $1, x, x^2, x^{3/2}, \sqrt{x}, 1/x, \sin x, \cos x, e^x, \ln x$ .

Combine 3 or more of the above functions, either by adding, subtracting, multiplying, dividing or composing. For example

- $\frac{e^{\sin x}}{\ln x}$
- $\sqrt{x^2 + \cos x}$
- $x^2 e^{\frac{1}{x}}$

Take the derivatives of the functions you generated using power rule, product rule, quotient rule and chain rule. Repeat as necessary.

2. Consider the function

$$f(x) = \ln x \cos(\pi x).$$

Find the equation of the line tangent to  $f(x)$  at the point  $x = 1$ .

3. Find all  $x$  values where  $f(x) = x^3 - 3x^2 - 9x + 14$  has horizontal tangent lines.
4. The radius of a circular ripple in the water is growing at a rate of 3 cm/s.
  - (a) How fast is the circumference of the circle growing when the radius is 10cm. Note that the formula for the circumference of a circle is

$$C = 2\pi r.$$

- (b) How fast is the area of the circle growing when the radius is 10cm. Note that the formula for the area of a circle is

$$A = \pi r^2.$$

5. You and your friend are doing something forestry related together after which you travel north and your friend travels west. Currently you are travelling at 100km/h and are 21km from where you started. Your friend is travelling at 90km/h and is 20km from where you met. How fast is the distance between you and your friend changing?
6. Approximate the following integral using Riemann Sums using right endpoints and  $n = 3$  (ie. 3 bars)

$$\int_1^{5/2} (5 - x) dx.$$

Is your approximation an over or under estimate (or exactly right)? Compute the integral to check. Explain how your approximation could be made better. Include pictures to demonstrate your point. Can your approximation scheme be rigged to give the area exactly?

7. Draw a function satisfying the following two properties:

- $f(2) = 4$
- $\int_0^4 f(x)dx = 0$ .

Explain how it is that there is 'zero' area under the curve when clearly the function is above the  $x$ -axis at  $x = 2$ .

8. Compute the following integrals using your preferred method.

- $\int_1^2 \left( \sqrt{x} - \frac{3}{\sqrt{x}} + 5x^2 \right) dx$
- $\int_{\ln 2}^{\ln 3} e^{-4x} dx$
- $\int_{\pi/2}^{\pi} \cos(3x + \pi) dx$
- $\int_2^5 xe^{-x} dx$

9. Find a function,  $f(x)$ , whose derivative is

$$f'(x) = x^2 + \sin(\pi x)$$

and also satisfies  $f(2) = 1/3$ .

10. Compute the following integrals. Use any methods at your disposal.

- $\int x \cos(3x) dx$
- $\int \frac{x}{(4-x^2)^2} dx$
- $\int e^x \sin(e^x) dx$
- $\int \sqrt{x} \ln x dx$
- $\int x\sqrt{3x^2+7} dx$
- $\int \sin^4 x \cos x dx$

11. A runner starts a race at time  $t = 0$ . After 1 second she has travelled 2 meters. Her velocity is given by

$$v(t) = x(6-x).$$

How far has she gone after 3 seconds (ie.  $t = 3$ ). By the way, velocity is the derivative of position.

12. As I blow up a balloon the surface area changes. The rate of change of the surface area is given by

$$r(t) = 1 + \cos(2t)$$

in units of  $\text{cm}^2/\text{second}$ . If after  $t = \pi/4$  seconds the balloon has a surface area of  $10 \text{ cm}^2$  find the surface area after  $t = \pi/2$  seconds.