

Before you look at these solutions ask yourself:

Did I try the problem for 10 minutes?

If not, try again.

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M190: Final Exam Practice

$$1 \quad \bullet \quad \left(\frac{e^{\sin x}}{\ln x} \right)' = \frac{\cos x e^{\sin x} \ln x - \frac{1}{x} e^{\sin x}}{(\ln x)^2}$$

$$\bullet \quad \left(\sqrt{x^2 + \cos x} \right)' = \frac{1}{2} (x^2 + \cos x)^{-1/2} \cdot (2x - \sin x)$$

$$\bullet \quad (x^2 e^{1/x})' = 2x e^{1/x} + x^2 e^{1/x} \left(\frac{-1}{x^2} \right)$$

$$2. \quad f(x) = \ln x \cos(\pi x)$$

$$f'(x) = \frac{1}{x} \cos(\pi x) - \pi \ln x \sin(\pi x)$$

$$f'(1) = \frac{1}{1} \cos(\pi) - \pi \ln(1) \sin(\pi)$$

$$= -1$$

$$y - y_0 = m(x - x_0)$$

$$m = -1$$

$$x_0 = 1$$

$$y = -1(x - 1)$$

$$y_0 = f(1) = \ln(1) \cos(\pi) = 0$$

$$3. \quad f(x) = x^3 - 3x^2 - 9x + 14$$

$$f'(x) = 3x^2 - 6x - 9$$

$$\text{Want } f'(x) = 0 = 3x^2 - 6x - 9$$

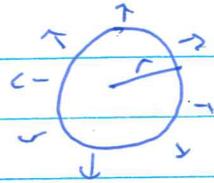
$$0 = 3(x^2 - 2x - 3)$$

$$0 = 3(x+1)(x-3)$$

$$\text{So, } x = -1, 3$$

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4. $\frac{dr}{dt} = 3 \text{ cm/s}$



a) $\frac{dC}{dt} = ?$

$\frac{dC}{dt} = 2\pi r \frac{dr}{dt}$

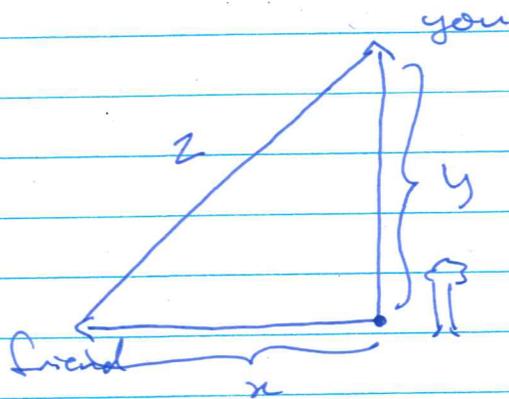
Chain Rule

$= 2\pi \cdot 3 \text{ cm/s}$

b)

$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt} = 2\pi (10) \cdot 3 \text{ cm}^2/\text{s}$

5.



Want: $\frac{dz}{dt}$ have: $\frac{dy}{dt} \leftarrow 90$ $\frac{dx}{dt} \leftarrow 100$

When $x=21, y=20$.

Eg. $z^2 = x^2 + y^2 \rightarrow z^2 = (21)^2 + (20)^2$
 $z = 29$

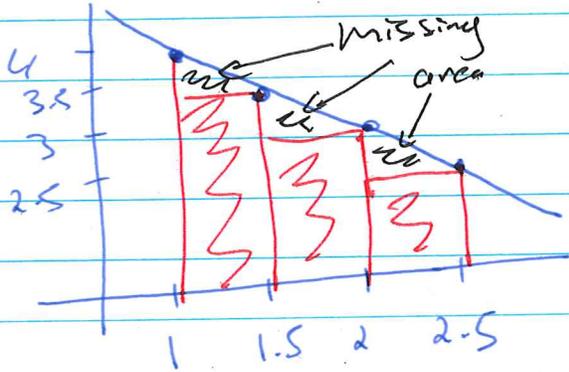
Chain Rule: $2z \frac{dz}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$

$\frac{dz}{dt} = \frac{1}{z} \left(x \frac{dx}{dt} + y \frac{dy}{dt} \right)$

$= \frac{1}{29} (21 \cdot 100 + 20 \cdot 90) \text{ km/h}$

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6. $\int_1^{5/2} (5-x) dx$



$$\sum_{i=1}^3 f(x_i) \Delta x$$

- $x_1 = 1.5 \quad f(x_1) = 3.5$
- $x_2 = 2 \quad f(x_2) = 3$
- $x_3 = 2.5 \quad f(x_3) = 2.5$

$\Delta x = 1/2$

$$\sum_{i=1}^3 f(x_i) \Delta x = \frac{1}{2} \cdot 3.5 + \frac{1}{2} \cdot 3 + \frac{1}{2} \cdot 2.5$$

$$= \frac{1}{2} (3.5 + 3 + 2.5)$$

$$= \frac{1}{2} \cdot 6 = 3$$

~~Over~~ Underestimate.

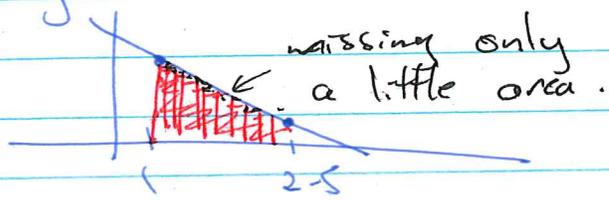
Observe from the picture that we have missed some area.

Check: $\int_1^{5/2} (5-x) dx = \left[5x - \frac{x^2}{2} \right]_{x=1}^{x=5/2}$

$$= 5 \cdot \frac{5}{2} - \frac{1}{2} \left(\frac{5}{2} \right)^2 - 5 + \frac{1}{2}$$

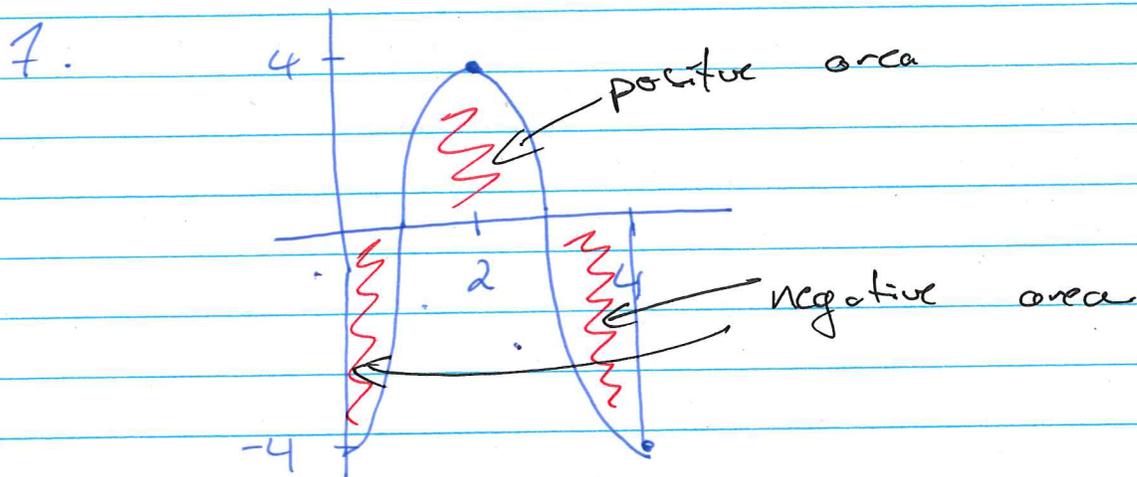
$$= 4.875 > 3$$

The approximation can be made better by increasing the number of bars (n).



If we take $\lim_{n \rightarrow \infty}$ then we will achieve the area exactly.

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8.

$$\int_1^2 \left(\sqrt{x} - \frac{3}{\sqrt{x}} + 5x^2 \right) dx$$

$$= \frac{2x^{3/2}}{3/2} - 3 \cdot 2(x)^{1/2} + \frac{5}{3} x^3 \Bigg|_{x=1}^{x=2}$$

$$= \frac{2}{3} 2^{3/2} - 6 \cdot 2^{1/2} + \frac{5(2)^3}{3}$$

$$- \frac{2}{3} + 3 \cdot 2 - \frac{5(1)}{3}$$

$$\int_{\ln 2}^{\ln 3} e^{-4x} dx = \frac{-e^{-4x}}{4} \Bigg|_{\ln 2}^{\ln 3}$$

or let $u = -4x$
 $du = -4dx$

$$= \frac{-1}{4} \left(e^{-4 \ln 3} - e^{-4 \ln 2} \right)$$

$$= \frac{-1}{4} (3^{-4} - 2^{-4})$$

$$\int_{\pi/2}^{\pi} \cos(3x + \pi) dx = \frac{1}{3} \sin(3x + \pi) \Bigg|_{\pi/2}^{\pi}$$

or let $u = 3x + \pi$
 $du = 3dx$

$$= \frac{1}{3} \sin(4\pi) - \frac{1}{3} \sin\left(\frac{5\pi}{2}\right)$$

$$= \frac{1}{3} \cdot 0 - \frac{1}{3} \cdot 1 = -\frac{1}{3}$$

③

$$\int_2^5 x e^{-x} dx \quad \left(\begin{array}{l} \text{let } u=x \quad dv=e^{-x} dx \\ du=dx \quad v=-e^{-x} \end{array} \right)$$

$$= -x e^{-x} \Big|_2^5 + \int_2^5 e^{-x} dx$$

$$= -x e^{-x} \Big|_2^5 - e^{-x} \Big|_2^5$$

$$= -5e^{-5} + 2e^{-2} - e^{-5} + e^{-2}$$

$$= -6e^{-5} + 3e^{-2}$$

9. $f'(x) = x^2 + \sin(\pi x)$
 $f(x) = \frac{x^3}{3} - \frac{1}{\pi} \cos(\pi x) + C$

$$f(2) = 1/3 = \frac{8}{3} - \frac{1}{\pi} \cos(2\pi) + C$$

$$C = -\frac{7}{3} + 1/\pi$$

$$f(x) = \frac{x^3}{3} - \frac{1}{\pi} \cos(\pi x) - \frac{7}{3} + 1/\pi$$

10.

$$\int x \cos(3x) dx \quad \left(\begin{array}{l} \text{let } dv = \cos(3x) dx \\ v = \frac{1}{3} \sin(3x) \end{array} \right)$$

$$= \frac{x}{3} \sin(3x) - \frac{1}{3} \int \sin(3x) dx$$

$$= \frac{x}{3} \sin(3x) + \frac{1}{9} \cos(3x) + C \quad \left(\begin{array}{l} u=x \\ du=dx \end{array} \right)$$

$$\int \frac{x}{(4-x^2)^2} dx = \frac{-1}{2} \int \frac{1}{u^2} du = \frac{1}{2} \frac{1}{u} + C$$

$$= \frac{1}{2(4-x^2)} + C$$

let $u=4-x^2$
 $du = -2x dx$

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• $\int e^x \sin(e^x) dx$ let $u = e^x$
 $du = e^x dx$

$= \int \sin(u) du$

$= -\cos u + C = -\cos(e^x) + C$

• $\int \sqrt{x} \ln x dx$ let $u = \ln x$ $dv = x^{1/2} dx$
 $du = \frac{1}{x} dx$ $v = \frac{2x^{3/2}}{3}$

$= \frac{2}{3} x^{3/2} \ln x - \frac{2}{3} \int x^{3/2} \frac{1}{x} dx$

$= \frac{2}{3} x^{3/2} \ln x - \frac{2}{3} \int x^{1/2} dx$

$= \frac{2}{3} x^{3/2} \ln x - \frac{2}{3} \cdot \frac{2}{3} x^{3/2} + C$

• $\int x \sqrt{3x^2+7} dx$ let $u = 3x^2+7$
 $du = 6x dx$

$= \frac{1}{6} \int \sqrt{u} du$

$= \frac{1}{6} \cdot \frac{2}{3} u^{3/2} + C = \frac{1}{9} (3x^2+7)^{3/2} + C$

• $\int \sin^4 x \cos x dx$ let $u = \sin x$
 $du = \cos x dx$

$= \int u^4 du = \frac{u^5}{5} + C = \frac{\sin^5 x}{5} + C$

II. $v(t) = 6t - t^2$ *should be t's.*

$s(t) = \int v(t) dt = \int (6t - t^2) dt = 3t^2 - \frac{t^3}{3} + C$

$s(1) = 2 = 3 - \frac{1}{3} + C \Rightarrow s(t) = 3t^2 - \frac{t^3}{3} - \frac{2}{3}$ $18 - 2(3)$

$\Rightarrow C = -\frac{2}{3}$ $s(3) = 3 \cdot 3^2 - \frac{3^3}{3} - \frac{2}{3}$

⑦

$$12. \quad r(t) = 1 + \cos(2t)$$

$$A(t) = \int 1 + \cos(2t) dt$$
$$= t + \frac{1}{2} \sin(2t) + C$$

$$A(\pi/4) = 10 = \pi/4 + \frac{1}{2} \sin(\pi/2) + C$$

$$C = 10 - \pi/4 - 1/2$$

$$A(t) = t + \frac{1}{2} \sin(2t) + 10 - \pi/4 - 1/2$$

$$A(\pi/2) = \pi/2 + \frac{1}{2} \sin(\pi) + 10 - \pi/4 - 1/2$$
$$= 10 + \pi/4 - 1/2$$